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Radial structures and nonlinear excitation of geodesic acoustic modes

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Abstract – Geodesic acoustic modes (GAM) are shown to constitute a continuous spectrum due to radial inhomogeneities. The importance and theoretical as well as experimental implications of this fact are discussed in this work. The existence of a singular layer causes GAM to mode convert to short-wavelength kinetic GAM (KGAM) via finite ion Larmor radii; analogous to kinetic Alfvén waves (KAW). Furthermore, it is shown that KGAM can be nonlinearly excited by drift-wave (DW) turbulence via 3-wave parametric interactions, and the resultant driven-dissipative nonlinear system exhibits typical prey-predator self-regulatory dynamics, consistent with recent experimental observations on HL-2A. The degeneracy of GAM/KGAM with beta-induced Alfvén eigenmodes (BAE) is demonstrated and discussed, with emphasis on its important role in the complex self-organized behaviors of burning plasmas.

The notion that toroidally and poloidally symmetric structures play a crucial role in the fundamental processes underlying turbulent transport in toroidal fusion devices is generally accepted. Particular examples of such structures are zonal flows (ZFs) [1], zonal fields [2–4], and radial corrugations of equilibrium profiles [5,6]. These zonal structures are important for the overall burning plasma performance and can be viewed as generators of nonlinear equilibria [7], whose stability determines the dynamics underlying their dissipation in collisionless plasmas and, ultimately, their interplay with turbulent transport.

Geodesic acoustic modes (GAM) [8], as finite-frequency counterpart of ZFs, are one particular kind of zonal structures, which can scatter drift-wave (DW) fluctuations to stable short-wavelength domain and, thereby, suppress the DW turbulence transport [9]. They have been extensively investigated in recent experiments by means of a variety of techniques: Beam Emission Spectroscopy (BES) on DIII-D [10], Heavy Ion Beam Probe (HIBP) on TEXT [11], CHS [12], T10 [13] and JFT-2M [14], Doppler reflectometry on Asdex Upgrade [15], multiple Langmuir probes on T10 [13] and HL-2A [16]. Although experimental evidence generally shows an inverse relation between the GAM intensity and the background fluctuation level, observations have not clarified yet the mechanism underlying GAM excitation, which theory identifies in the ambient turbulence [9,17,18].

In this paper, we show that GAM [8] constitute a continuous spectrum due to radial inhomogeneities with a one-to-one analogy with the shear Alfén wave (SAW) continuum. The existence of a continuous spectrum has important consequences on the plasma dynamics as well as on experimental observations. Satellite observations of the SAW continuum in the Earth magnetosphere are common [19]; meanwhile GAM frequency dependence on the radial position has also been reported [13]. The local (singular) fluctuations of which a continuous spectrum is constituted decay in time as t^{-1} due to phase mixing [20]; so, they are observable only if excited by an external source, which influences the manifestation and detection of the fluctuations themselves, as discussed below.

The existence of a GAM continuum and, thus, of a singular layer, suggests linear mode conversion to short-wavelength kinetic GAM (KGAM) via finite ion Larmor radii. This result is demonstrated here by derivations of the GAM mode structure and dispersion relation in the

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singular layer. At the lowest order in $k_\zeta \rho_i$, with k_ζ the radial wave vector, $\rho_i = v_{ti}/\omega_{ci}$ the ion Larmor radius, $v_{ti} = (2T_i/m_i)^{1/2}$ and $\omega_{ci} = (eB)/(m_i c)$, the well-known kinetic dispersion relation of GAM is recovered [21,22]. At the next relevant order, $O(k_\zeta^2 \rho_i^2)$, we show that KGAM typically propagates in the low-temperature and/or high–safety-factor domain; *i.e.*, radially outward. Our analyses also confirm that GAM and beta-induced Alfvén eigenmodes (BAE) [23,24] are degenerate in the long-wavelength limit, when diamagnetic effects are ignored, even when finite Larmor radius corrections are accounted for. As reported earlier [6,25,26], this is not a coincidence but a consequence of the fact that SAW compressibility due to geodesic curvature coupling at $k_\parallel = 0$ is identical to the corresponding dynamics of electrostatic waves with $k_\phi = k_\theta = 0$. The BAE/GAM degeneracy is expected to play an important role in the complex self-organized behaviors of burning plasmas since, given the very disparate space-time scales of Alfvénic fluctuations and MHD modes on the one side and plasma turbulence on the other, their nonlinear interplay via zonal structures is expected to be one of the dominant cross-scale coupling mechanisms [6]. In this work, we also analyze GAM/KGAM nonlinear dynamics and show that, while GAM/KGAM are linearly stable due to ion Landau damping, they can be nonlinearly excited by finite-amplitude DW turbulence via 3-wave resonant parametric interactions [17,18].

The BAE/GAM degeneracy can be easily demonstrated considering the magnetic flux surface averaged quasi-neutrality condition for axisymmetric fluctuations (toroidal mode number $n = 0$), which reads

$$\partial_r \left(\overline{\delta J_r} \right) = 0, \tag{1}$$

where δJ_r is the fluctuating radial current and $\overline{(\dots)}$ denotes magnetic flux surface averaging. Here, we have considered a general axisymmetric toroidal equilibrium with straight field line flux coordinates (r, θ, ξ) and the equilibrium magnetic field given by the Clebsch representation, $\mathbf{B} = \nabla(\xi - q\theta) \times \nabla\psi_p$, with $q(\psi_p) = \mathbf{B} \cdot \boldsymbol{\nabla}\xi / \boldsymbol{B} \cdot \nabla\theta = \mathrm{d}\psi/\mathrm{d}\psi_p$ and $\psi(\psi_p)$ the toroidal (poloidal) magnetic flux function. Equation (1) describes GAM as well as flute-like SAW, like BAE, near the $qR_0 k_\parallel = nq - m = 0$ surface, m being the poloidal mode number and R_0 the torus major radius. At $k_\parallel = 0$ and for $\omega_*/\omega \to 0$, with ω_* the diamagnetic frequency, the GAM and BAE dynamics must be identical since for both fluctuations the dynamic behavior is dominated by the particle response to the radial electric field, which reflects the particle radial magnetic drifts associated with geodesic curvature. The BAE/GAM degeneracy has been recently noted in [27] as well. Here, it is worthwhile emphasizing further that the implication of eq. (1) being identical for $n = m = 0$ modes and flute-like SAW near $qR_0 k_\parallel = nq - m = 0$ is far more general than proving the BAE/GAM degeneracy: it also proves that singular layer inertial response of SAW and MHD modes at low frequency ($|\omega R_0/v_{ti}| \ll 1$) is identical to the ZF polarizability induced by ion temperature gradient (ITG) turbulence [28,29], as explicitly shown in [26].

Besides the importance of its physics implications, the usefulness of this result on the BAE/GAM degeneracy is that we may straightforwardly derive the governing equations for GAM using the kinetic theory results on BAE developed earlier [30–32]. Defining $\Omega \equiv (\omega/\omega_{ti})$, with $\omega_{ti} = v_{ti}/(qR_0)$ the ion transit frequency, and using eqs. (12) and (14) of [30], we readily cast eq. (1) above in the following form:

$$\partial_r \left(N_0 \Lambda_0^2(\Omega) \partial_r \delta\phi \right) = 0, \tag{2}$$

$$\Lambda_0^2(\Omega) = 1 + \frac{q^2}{\Omega} \left(F(\Omega) - \frac{N^2(\Omega)}{D(\Omega)} \right). \tag{3}$$

Here, $N_0 = N_0(r)$ is the plasma density, which we assumed the same for electrons and unit charge ions, Λ_0^2 is the Λ^2 function introduced in [30,33] evaluated at $\omega_*/\omega = 0$ and renormalized by a factor $v_A^2/(qR_0\omega)^2$, with v_A the Alfvén speed. Meanwhile, $F(\Omega) = \Omega(\Omega^2 + 3/2) + (\Omega^4 + \Omega^2 + 1/2) Z(\Omega)$, $N(\Omega) = \Omega + (1/2 + \Omega^2) Z(\Omega)$, $D(\Omega) = Z(\Omega) + (1 + T_i/T_e)(1/\Omega)$ and $Z(\Omega) \equiv \pi^{-1/2} \int_{-\infty}^{\infty} \mathrm{e}^{-y^2}/(y - \Omega)\mathrm{d}y$ is the plasma dispersion function. Equation (3), based on the results of [30] and on the proof that BAE and GAM spectra are degenerate for $\omega_*/\omega \to 0$ [6,25,26], is valid in the $k_\zeta \rho_i q \ll 1$ limit and coincides with the corresponding expressions given by Sugama *et al.* [34] and by Gao *et al.* [35] in the $T_e/T_i = 0$ limit. From eq. (3), it is readily verified that Λ_0 depends on $T_e(r)$, $T_i(r)$ and $q(r)$, which are all functions of the radial position. Equation (2) is similar to that describing the SAW resonance [36] and, thus, demonstrates that GAM constitutes a continuous spectrum described by $\Lambda_0^2 = 0$, giving the solution $\omega = \omega_{GAM}(r)$. In particular, fluctuations of the GAM continuous spectrum consist of singular structures, whose time asymptotic behavior is quasi-exponential [37,38] $\propto (1/t)\exp(-i\omega_{GAM}(r)t)$. The singular nature of the fluctuations is embedded in the corresponding value of k_ζ, which increases in time as [39]

$$k_\zeta \simeq -(\mathrm{d}\omega_{GAM}(r)/\mathrm{d}r)t. \tag{4}$$

Equation (4) can be viewed as a physical manifestation of phase mixing [20] of fluctuations belonging to the GAM continuous spectrum and, as such, it is an observable phenomenon: see *e.g.*, fig. 4 of ref. [40] for a visualization of this effect from numerical simulations. Experimentally, when the system is globally perturbed with a broad-band frequency spectrum at some initial time, phase mixing as described by eq. (4) should be visible as radial spreading of the fluctuations belonging to the continuous spectrum with characteristic speed scaling as the phase velocity [41]. When $k_\zeta \rho_i q \sim 1$ additional phenomena start becoming important as discussed below. Thus, characteristic times on which observations are made are a crucial information for comparisons with theoretical predictions.

In the fluid limit, $|\Omega| = |\omega/\omega_{ti}| \gg 1$, the large-argument expansion of $Z(\Omega)$ in eq. (3) yields [30]

$$\Lambda_0^2 \simeq 1 - \frac{q^2}{\Omega^2}\left(\frac{7}{4} + \frac{T_e}{T_i}\right) + i\pi^{1/2} q^2 \Omega^3 e^{-\Omega^2}. \qquad (5)$$

Solving $\Lambda_0^2 = 0$ with eq. (5) recovers the well-known kinetic expression of the GAM frequency [21,22] and the expression of the corresponding Landau damping. Equation (5) can also be recovered from ref. [42], invoking the degeneracy of BAE and GAM spectra [6,25,26]. Retaining one order higher in the large-argument expansion of $Z(\Omega)$ in eq. (3) modifies Λ_0^2 in eq. (5) into [26,30]

$$\Lambda_0^2 \simeq 1 - \frac{q^2}{\Omega^2}\left(\frac{7}{4} + \frac{T_e}{T_i}\right) - \frac{q^2}{\Omega^4}\left(\frac{23}{8} + \frac{2T_e}{T_i} + \frac{T_e^2}{2T_i^2}\right) + i\pi^{1/2} q^2 \Omega^3 e^{-\Omega^2}\left[1 + (1 + 2T_e/T_i)/\Omega^2\right]. \qquad (6)$$

This expression agrees with those given by Gao *et al.* [35] in the $T_e/T_i = 0$ limit and by Sugama *et al.* [43]. Note that eq. (6) demonstrates that the real GAM frequency does not contain the $1 + (2q^2)^{-1}$ Pfirsch-Schlüter factor predicted by MHD [8]. Note also that eqs. (5) and (6) represent fairly crude approximations of Λ_0^2, unless $q^2(7/4 + T_e/T_i) \gg 1$; thus, when computing GAM frequency and damping rate from $\Lambda_0^2 = 0$, the more precise eq. (3) should be used, especially when comparing theory with experimental observations.

When finite Larmor radius (FLR) and finite orbit width (FOW) effects are included assuming $k_\zeta \rho_i q \ll 1$, Λ_0^2 in eq. (2) is replaced by

$$\Lambda_0^2 \to \Lambda_0^2 - (k_\zeta^2 \rho_i^2/2)\left(3/4 + (q^2/\Omega) S_0(\Omega)\right), \qquad (7)$$

where $S_0(\Omega)$ is the function $S(\Omega)$, defined in eq. (B28) of [31] and eq. (21) of [32], evaluated at $\omega_*/\omega = 0$, *i.e.*

$$S_0(\Omega) = \frac{q^2}{2\Omega^2}\left[L(\Omega) - 2L\left(\frac{\Omega}{2}\right) - \frac{2N(\Omega)}{D(\Omega)}\left(H(\Omega) - 2H\left(\frac{\Omega}{2}\right)\right) + \frac{N(\Omega)^2}{D(\Omega)^2}\left(F - 2F\left(\frac{\Omega}{2}\right)\right)\right] + T(\Omega) - \frac{2N(\Omega)}{D(\Omega)} V(\Omega) + \frac{N(\Omega)^2}{D(\Omega)^2} Z(\Omega) + \frac{q^2}{\Omega^2 D(\Omega/2)}\left[F\left(\frac{\Omega}{2}\right) - F(\Omega) - \frac{N(\Omega)}{D(\Omega)}\left(N\left(\frac{\Omega}{2}\right) - N(\Omega)\right)\right]^2, \qquad (8)$$

with $L(\Omega) = \Omega^7 + (5/2)\Omega^5 + (19/4)\Omega^3 + (63/8)\Omega + (\Omega^8 + 2\Omega^6 + 3\Omega^4 + 3\Omega^2 + 3/2) Z(\Omega)$, $H(\Omega) = \Omega^5 + 2\Omega^3 + 3\Omega + (\Omega^6 + (3/2)\Omega^4 + (3/2)\Omega^2 + 3/4) Z(\Omega)$, $T(\Omega) = \Omega^3 + (5/2)\Omega + (\Omega^4 + 2\Omega^2 + (3/2)) Z(\Omega)$ and $V(\Omega) = \Omega + (\Omega^2 + 1) Z(\Omega)$. In the fluid limit, $|\Omega| = |\omega/\omega_{ti}| \gg 1$, from the expression of $S_0(\Omega)$ [31,32] one can readily show $(q^2(7/4 + T_e/T_i) \gg 1)$

$$\frac{3}{4} + \frac{q^2}{\Omega} S_0(\Omega) \simeq \frac{3}{4} - \frac{q^2}{\Omega^2}\left(\frac{13}{4} + 3\frac{T_e}{T_i} + \frac{T_e^2}{T_i^2}\right) + \frac{q^4}{\Omega^4}\left(\frac{747}{32} + \frac{481}{32}\frac{T_e}{T_i} + \frac{35}{8}\frac{T_e^2}{T_i^2} + \frac{1}{2}\frac{T_e^3}{T_i^3}\right) - i\pi^{1/2} q^4 e^{-\Omega^2/4}\left[\Omega^5/256 + (1 + T_e/T_i)\Omega^3/32\right]. \qquad (9)$$

Note that the $\propto \exp(-\Omega^2/4)$ indicates the dominant role of the $|\omega| = 2\omega_{ti}$ resonance in $S_0(\Omega)$ [31,32] for determining the GAM damping. The imaginary part of eq. (9) coincides with the expression given in [43] and confirms that FLR/FOW effects strengthen GAM collisionless dissipation [34]. The real part, meanwhile, is consistent with the expression presented recently by Nguyen *et al.* [44].

With the prescription of eq. (7), the structure of eq. (2) is identical to that describing SAW mode conversion to KAW near the SAW resonance [45]. Thus GAM mode conversion to short-wavelength KGAM is expected near the singular layer ($\Lambda_0^2 = 0$), with the well-known Airy function behavior for the homogeneous solution of the modified eq. (2) [45]. For $|\Omega| = |\omega/\omega_{ti}| \gg 1$, eq. (9) shows $\mathbb{R}\text{e}[3/4 + (q^2/\Omega) S_0] > 0$, *i.e.* that KGAM has a wave-like radial structure [17,27,46] and is propagating in the high-frequency region ($\mathbb{R}\text{e}\Lambda_0^2 > 0$); *i.e.*, typically, radially outward, in the low-temperature and/or high–safety-factor domain [17,27,46], consistent with experimental observations [41,47–49] and simulation results [50,51]. Incidentally, we note that $\mathbb{R}\text{e}[3/4 + (q^2/\Omega) S_0]$ can be negative for moderate q^2. More precisely, numerical evaluation of $S_0(\Omega)$ shows that, for $q < 2.6$, it is possible to define a critical $(T_e/T_i)_{in}$ value below which KGAM propagates radially inward; for $q > 2.6$ KGAM always propagates radially outward. A fit of numerical results gives $(T_e/T_i)_{in} \simeq -2.62 + 26.87/q^2 - 80.11/q^4 + 125.4/q^6$. Inward-propagating KGAM have been observed experimentally [48], although, for propagation in the low-frequency region ($\mathbb{R}\text{e}\Lambda_0^2 < 0$) at high-temperature and/or moderate safety-factor, the mode is expected to be strongly Landau damped (see eqs. (5)–(9) and (14)) and turbulence is predominantly regulated by zero-frequency ZF [4,9].

The Airy function behavior for the homogeneous solution of eq. (2), modified by FLR/FOW [45] as prescribed by eq. (7), does not allow solving for the GAM/KGAM frequency, which remains undetermined until the non-homogeneous problem is solved in the presence of a source term [45]. The source term can be associated with either an anisotropic distribution of (fast) particles in velocity space [27,52] or by nonlinear excitations due to DW turbulence [9]. In either case, global sources that excite the system with a broad-band frequency spectrum at some initial time tend to excite the GAM continuum, while a more localized source with a narrow frequency spectrum tends to excite KGAM. The time coherence of the source is therefore an important factor as well, since it introduces some characteristic time and corresponding frequency in GAM/KGAM excitations.

Since nonlinear excitation favors short KGAM radial wavelengths (see below), we need to relax the $k_\zeta \rho_i q \ll 1$ assumption in eqs. (3) to (9) and derive corresponding expressions that are valid at short wavelength. For $1 \gg k_\zeta \rho_i \gg (1/q) \sim (k_\zeta \rho_i)^2$, we can solve the quasi-neutrality condition

$$(e/T_e)\left(\delta\Phi_k - \overline{\delta\Phi_k}\right) = -(e/T_i)\delta\Phi_k + \langle J_k \delta g_k \rangle / N_0, \qquad (10)$$

via asymptotic expansion solutions of the linear gyrokinetic equation for the non-adiabatic ion response δg_k. Here, the GAM/KGAM scalar potential fluctuation $\delta\Phi_k$, at the leading order, is given by

$$\delta\Phi_k \simeq \delta\Phi_\zeta = A_\zeta e^{ik_\zeta r - i\omega_\zeta t} + \text{c.c.}, \tag{11}$$

$J_k \equiv J_0(k_\perp v_\perp/\omega_{ci})$, $\langle \ldots \rangle$ indicates velocity space integration and δg_k is the solution of

$$\mathcal{L}_g^\ell \delta g_k = \left(\partial_t + v_\parallel \partial_\parallel + i\omega_d\right)\delta g_k = (e/T_i)F_0 J_k \partial_t \delta\Phi_k, \tag{12}$$

where $\omega_d \simeq \hat{\omega}_d \sin\theta = -k_\zeta \rho_i (v_{ti}/R_0)(v_\perp^2/2 + v_\parallel^2)/v_{ti}^2 \sin\theta$ is the magnetic drift frequency and F_0 is the ion equilibrium distribution function. Up to second order in the $\omega_d/\omega \sim k_\zeta\rho_i \sim 1/q^{1/2}$ asymptotic expansion, we have

$$\delta g_k = (e/T_i)F_0 \Big[\left(J_\zeta + \omega_d/\omega + \omega_d^2/\omega^2\right)\delta\Phi_\zeta + \delta\tilde{\Phi} + \omega_d \delta\tilde{\Phi}/\omega - i\pi\omega\delta(\omega-\omega_d)(\delta\Phi_\zeta + \delta\tilde{\Phi})\Big], \tag{13}$$

where $J_\zeta = J_0(k_\zeta v_\perp/\omega_{ci})$ and $\delta\tilde{\Phi} = \delta\Phi - \delta\Phi_\zeta \simeq -T_e/T_i \times (\omega_{dti}/\omega)\left[\sin\theta + (7/8 + T_e/2T_i)\omega_{dti}\cos 2\theta/\omega\right]\delta\Phi_\zeta$, with $\omega_{dti} = k_\zeta\rho_i(v_{ti}/R_0)$. Given eq. (13), one readily derives the GAM/KGAM dispersion relation in the form

$$\epsilon_g = 1 - \left\langle \left[J_\zeta^2 + \hat{\omega}_d^2/(2\omega^2)\right] F_0/N_0 \right\rangle - (T_e/2T_i)\omega_{dti}^2/\omega^2 + \sqrt{2} i\sigma \text{sgn}(\omega_{dti}) e^{-\sigma\omega/\omega_{dti}} \left(1 + \frac{T_e/T_i}{\sigma\omega/\omega_{dti}}\right) = 0, \tag{14}$$

where ϵ_g is the effective GAM/KGAM dielectric function and $\sigma = \text{sgn}(\omega/\omega_{dti})$. From eq. (14), the corresponding eq. (2) is readily derived for $1 \gg k_\zeta\rho_i \gg (1/q)$ [17] by noting the equivalence $\epsilon_g \equiv (k_\zeta^2\rho_i^2/2)\Lambda_0^2$. The real GAM/KGAM frequencies predicted by eqs. (5) and (14) are the same, as expected, while damping is underestimated by eq. (5) for large q, the transition from eq. (5) to (14) damping occurring at $k_\zeta\rho_i q^2 \sim 1$, since $\omega/\omega_{dti} = \Omega/(k_\zeta\rho_i q)$. Proceeding further up to the fourth order in the ω_d/ω asymptotic expansion of δg_ζ, we can compute the FLR/FOW corrections to ϵ_g and find $\epsilon_g \to (k_\zeta^2\rho_i^2/2)\Lambda_0^2 - (k_\zeta^2\rho_i^2/2)^2\lambda$. Since the present ordering $1 \gg k_\zeta\rho_i \gg (1/q)$ and the fluid limit of eqs. (3), (5) and (9) all refer to $k_\zeta\rho_i \ll 1$, one finds $\mathbb{R}\text{e}\lambda = \text{Re}[3/4 + (q^2/\Omega)S_0]$, given by eq. (9), as expected. Thus, in the short-wavelength (large q) limit $1 \gg k_\zeta\rho_i \gg (1/q) \sim (k_\zeta\rho_i)^2$, KGAM always propagate outward, consistently with eq. (9) and the following discussions as well as with most common experimental evidence [41,47–49]. Detailed discussion of higher-order corrections to $\mathbb{I}\text{m}\epsilon_g$ with respect to eq. (14) will be given in another work along with comparisons of analytic evaluations of GAM/KGAM damping *vs.* recent numerical-simulation results [51]. Here, we want to stress that the occurrence of the $\propto \delta(\omega - \omega_d)$ term in eq. (13) indicates that all transit resonance harmonics – $\omega = \ell\omega_{ti}$, $\ell = 1, 2, 3, \ldots$ – must contribute to GAM/KGAM damping for $q \gg 1$, as pointed out in [35]; the advantage of eq. (14) is the very simple expression of $\mathbb{I}\text{m}\epsilon_g$ that can be obtained for $|\omega_d/\omega_{ti}| \sim k_\zeta\rho_i q \gg 1$ and compares well with numerical simulation results [51]. Furthermore, the present analytic result is not limited to the case $T_e/T_i = 0$.

As pointed out above, GAM [8] are important to turbulence transport studies, since their low-frequency radial structures can scatter DW fluctuations to stable short-wavelength domain and, thereby, suppress the DW turbulence transport [9]. In this work, we show that, while GAM/KGAM are linearly stable due to ion Landau damping, they can be nonlinearly excited by finite-amplitude DW turbulence via 3-wave resonant parametric interactions. Nonlinear GAM excitation by DWs via 3-wave resonant interactions was recently discussed in [18] using a fluid model. Unlike [18], we use kinetic theory throughout, since it is crucial in order to account for DW sideband and KGAM damping at short wavelengths and for the determination of the KGAM excitation threshold. Nonlinear excitations of GAMs by DWs using the wave-kinetic approach have been investigated in [9,18].

Here, we follow the approach of ref. [53] and assume that a pump wave in DW turbulence spectrum (*e.g.* an ITG mode) is characterized by frequency ω_0 and wave-vector $\mathbf{k}_0$, while the corresponding scalar potential in toroidal geometry is

$$\delta\Phi_0 = A_0 e^{-in_0\zeta} \sum_m e^{im\theta - i\omega_0 t}\Phi_0(n_0 q - m) + \text{c.c.}, \tag{15}$$

where A_0 is the mode amplitude and $\Phi_0(n_0q - m)$ provides the radial structure of the single poloidal harmonics m. In the following, we demonstrate that the pump DW can spontaneously decay into a zonal mode (KGAM), given by eq. (11) and characterized by $(\omega_\zeta, \mathbf{k}_\zeta)$, and a lower-sideband DW (ITG)

$$\delta\Phi_- = A_- e^{in_0\zeta + ik_\zeta r - i\omega_- t} \sum_m e^{-im\theta}\Phi_0^*(n_0 q - m) + \text{c.c}, \tag{16}$$

with $\omega_- = \omega_\zeta - \omega_0^*$ and $\mathbf{k}_0 + \mathbf{k}_- = \mathbf{k}_\zeta = \hat{\mathbf{r}}k_\zeta$.

The nonlinear gyrokinetic equation [54] can be formally solved for the non-adiabatic ion response as $\delta g_\zeta = \delta g_\zeta^\ell + \delta g_\zeta^{n\ell}$, with δg_ζ^ℓ the well known linear response, given by eq. (13), and

$$\mathcal{L}_g^\ell \delta g_\zeta^{n\ell} = -\overline{\left[\delta\mathbf{u}_{E0}\cdot\nabla\delta g_- + \delta\mathbf{u}_{E-}\cdot\nabla\delta g_0\right]}, \tag{17}$$

with $\delta\mathbf{u}_{Ek} = i(c/B)(\mathbf{b}\times\mathbf{k}_\perp)J_k\delta\Phi_k$ and $\mathcal{L}_g^\ell \delta g_\zeta^{n\ell} \simeq \partial_t \delta g_\zeta^{n\ell}$. The nonlinear quasi-neutrality condition for the zonal mode then is

$$\partial_t\left[(e/T_i)A_\zeta - \left\langle J_\zeta \delta g_\zeta^\ell\right\rangle\right] = \left\langle (J_\zeta/N_0)\partial_t \delta g_\zeta^{n\ell}\right\rangle = -\left\langle (J_\zeta/N_0)\overline{\left[\delta\mathbf{u}_{E0}\cdot\nabla\delta g_- + \delta\mathbf{u}_{E-}\cdot\nabla\delta g_0\right]}\right\rangle. \tag{18}$$

Following ref. [53] and averaging on the fast radial variations $\propto |\Phi_0(n_0q - m)|^2$, associated with the local structures of the DW poloidal harmonics, eq. (18) becomes

$$\frac{\partial}{\partial t}\epsilon_g A_\zeta = -\frac{c}{2B}\alpha_i k_\theta k_\zeta k_\zeta^2\rho_i^2 \left\langle\left\langle |\hat{\Phi}_0|^2 \right\rangle\right\rangle A_0 A_-, \tag{19}$$

where [53] $\alpha_i = \delta P_{\perp i0}/(eN_0\delta\phi_0)+1$, $\delta P_{\perp i0}$ is the perpendicular ion pressure fluctuation due to the pump DW (ITG) and $\langle\langle|\hat{\Phi}_0|^2\rangle\rangle = \sum_m \int_{m-1/2}^{m+1/2} |\Phi_0(n_0q-m)|^2 \mathrm{d}(n_0q) = \int_{-\infty}^{+\infty} |\hat{\Phi}_0(\eta)|^2 \mathrm{d}\eta$, with $\hat{\Phi}_0(\eta) \equiv (2\pi)^{-1/2} \times \int_{-\infty}^{+\infty} \exp[i(n_0q-m)\eta]\Phi_0(n_0q-m)d(n_0q-m)$ the Fourier representation of $\Phi_0(n_0q-m)$.

Given the pump DW (ITG) and the zonal mode (KGAM), the lower-sideband DW (ITG) obeys the same evolution equation discussed in ref. [53], *i.e.*

$$D_- A_- = \frac{i}{\omega_0}\frac{c}{B}k_\theta k_\zeta \frac{T_i}{T_e} A_\zeta A_0^*, \tag{20}$$

with D_- the sideband dispersion function

$$D_- \simeq (\partial D_{0r}/\partial\omega_0)\left(\Delta - \omega_\zeta - i\gamma_d\right). \tag{21}$$

Here, D_{0r} is the Hermitian part of the DW (ITG) dispersion function, γ_d is the sideband damping, $\Delta = (k_\zeta^2/2)(\partial D_{0r}/\partial\omega_0)^{-1}(\partial^2 D_{0r}/\partial k_{0r}^2) = \omega_0 - \omega_1$ and ω_1 is the solution of $D_{0r}(\omega_1, \mathbf{k}_{0\theta} \pm \mathbf{k}_\zeta) = 0$ [53]. From eqs. (19) and (21), the frequency resonance condition for the resonant decay gives

$$\begin{cases} \omega_\zeta = \omega_{gr} + i\gamma_\zeta, \\ \omega_{gr} = \Delta, \end{cases} \tag{22}$$

where the real KGAM zonal mode frequency, ω_{gr}, is such that $\epsilon_{gr}(\omega_{gr}) = 0$. Given eq. (22), we readily have $\epsilon_g \simeq (i/\omega_{gr})k_\zeta^2\rho_i^2(\gamma_\zeta + \gamma_g)$, with γ_g the KGAM damping, and $D_- \simeq -i(\partial D_{0r}/\partial\omega_0)(\gamma_\zeta + \gamma_d)$ from eq. (21). In this way, eqs. (19) and (20) become, respectively,

$$(\gamma_\zeta + \gamma_g) A_\zeta = -\frac{c}{2B}\alpha_i k_\theta k_\zeta \left\langle\left\langle|\hat{\Phi}_0|^2\right\rangle\right\rangle A_0 A_-, \tag{23}$$

$$(\gamma_\zeta + \gamma_d) A_- = -\frac{c}{B}\frac{T_i/T_e}{\omega_0(\partial D_{0r}/\partial\omega_0)} k_\theta k_\zeta A_\zeta A_0^*. \tag{24}$$

Denoting $c_s^2 = T_e/m_i$, $\rho_s^2 = c_s^2/\omega_{ci}^2$ and

$$\gamma_{RD}^2 = \frac{\alpha_i(T_i/T_e)/2}{\omega_0(\partial D_{0r}/\partial\omega_0)}(k_\theta\rho_s k_\zeta c_s)^2 \left\langle\left\langle|\hat{\Phi}_0|^2\right\rangle\right\rangle \left|\frac{e}{T_e}A_0\right|^2, \tag{25}$$

from eqs. (23) and (24) we readily derive the excitation rate of the KGAM zonal mode, γ_ζ,

$$(\gamma_\zeta + \gamma_g)(\gamma_\zeta + \gamma_d) = \gamma_{RD}^2. \tag{26}$$

At threshold, eq. (26) gives $\gamma_{RD,th}^2 = \gamma_g\gamma_d$, while the wavelength of the KGAM can be estimated from eq. (22), *i.e.* $\omega_{gr} = \Delta \approx \omega_0(k_\zeta^2/k_{0r}^2)$, yielding $k_\zeta\rho_i \approx |\omega_{gr}/\omega_0|^{1/2}k_{0r}\rho_i$.

The result of eq. (26) should be compared with the zero-frequency ZF generation rate, Γ_ζ, given in ref. [53]

$$(\Gamma_\zeta + \nu_\zeta) = \gamma_M^2 \frac{(\Gamma_\zeta + \gamma_d)}{\Delta^2 + (\Gamma_\zeta + \gamma_d)^2}. \tag{27}$$

Here, ν_ζ is the ZF collisional dissipation rate [29], while [2,53]

$$\gamma_M^2 = \frac{(r/R_0)^{1/2}}{1.6q^2}\frac{2\alpha_i(T_i/T_e)}{\omega_0(\partial D_{0r}/\partial\omega_0)}(k_\theta\rho_s k_\zeta c_s)^2 \times \left\langle\left\langle|\hat{\Phi}_0|^2\right\rangle\right\rangle \left|\frac{e}{T_e}A_0\right|^2. \tag{28}$$

Significantly above threshold, $\gamma_\zeta \simeq \gamma_{RD}$ and $\Gamma_\zeta \simeq \gamma_M$. Thus, both γ_ζ and Γ_ζ scale linearly with A_0 and k_ζ. That nonlinear excitation favors zonal modes at short radial wavelengths justifies our suggestion that KGAM are preferentially excited by the 3-wave parametric interactions described here. Meanwhile, $\gamma_M^2/\gamma_{RD}^2 = 4(r/R_0)^{1/2}/(1.6q^2)$; *i.e.* the KGAM and zero-frequency ZF generation rates have similar scalings, so that their relative importance may be ultimately determined by the threshold condition, which for KGAM reads $\gamma_{RD,th}^2 = \gamma_g\gamma_d$ and was derived above, while for zero-frequency ZF is given by $\gamma_{M,th}^2 = (\nu_\zeta/\gamma_d)(\Delta^2 + \gamma_d^2)$ [53].

Denote $A_0 e^{-i\omega_0 t} = a_0(t)e^{-i\omega_0 t}$, $A_- e^{-i\omega_- t} = a_-(t)e^{i\omega_0 t - i\omega_{gr} t}$ and $A_\zeta e^{-i\omega_\zeta t} = a_\zeta(t)e^{-i\omega_{gr} t}$. Since ω_{gr} is independent of k_ζ in the lowest order, it is obvious to expect that k_ζ will have a toroidal mode number dependence via the wave frequency and number matching conditions; thus, $a_\zeta(t)e^{ik_\zeta r} = \sum_n a_{\zeta n}(t)e^{ik_{\zeta n} r}$. Similarly, the KGAM zonal mode damping will reflect the n dependence via $k_{\zeta n}$, typically increasing with $k_{\zeta n}\rho_i$, as suggested by eq. (14). In this way, we can rewrite the nonlinear dynamic system given by eqs. (23) and (24) as

$$(\partial_t + \gamma_{gn}) a_{\zeta n} = -\frac{c}{2B}\alpha_i k_{\theta n} k_{\zeta n} \left\langle\left\langle|\hat{\Phi}_0|^2\right\rangle\right\rangle a_{0n} a_{-n}, \tag{29}$$

$$(\partial_t + \gamma_{dn}) a_{-n} = -\frac{(c/B)(T_i/T_e)}{\omega_{0n}(\partial D_{0r}/\partial\omega_{0n})} k_{\theta n} k_{\zeta n} a_{\zeta n} a_{0n}^*, \tag{30}$$

$$(\partial_t - \gamma_{0n}) a_{0n} = \frac{(c/B)(T_i/T_e)}{\omega_{0n}(\partial D_{0r}/\partial\omega_{0n})} k_{\theta n} k_{\zeta n} a_{\zeta n} a_{-n}^*. \tag{31}$$

Here, we have written eq. (31) following the same derivation used for eq. (30) and γ_{0n} is the linear growth rate of the pump DW (ITG) with toroidal mode number n. In addition, we have considered all possible pump DW (ITG) toroidal mode numbers, extending eqs. (23) and (24). This driven-dissipative system based on 3-wave couplings exhibits limit-cycle behaviors, period-doubling and route to chaos as possible indication of the existence of strange attractors [55]. The 3-wave nonlinear system is characterized by prey-predator self-regulation. In fact, eqs. (29) to (31) obey the following (plasmon) energy conservation laws:

$$(\partial_t - 2\gamma_{0n})|a_{0n}|^2 = -(\partial_t + 2\gamma_{dn})|a_{-n}|^2, \tag{32}$$

$$(\partial_t - 2\gamma_{0n})|a_{0n}|^2 = -\frac{(2/\alpha_i)(T_i/T_e)}{\omega_{0n}(\partial D_{0r}/\partial\omega_{0n})}\left\langle\left\langle|\hat{\Phi}_0|^2\right\rangle\right\rangle^{-1} \times (\partial_t + 2\gamma_{gn})|a_{\zeta n}|^2. \tag{33}$$

These general properties are consistent with recent experimental observations on HL-2A [41], which show that electric field and density fluctuation radial envelopes are modulated by GAM via an energy-conserving triad interaction; this is further confirmed by cross- and auto-bicoherence analyses for interactions between GAM and turbulent fluctuations that reflect the resonant nature of GAM-DW nonlinear coupling [41].

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