Collisionless Damping of Short Wavelength Geodesic Acoustic Modes

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Motivation

- DW-GAM interaction
- Previous theoretical work

Present work

- linear gyrokinetic equation
- non-resonant ion response
- resonant ion response

Conclusion

DW-GAM interaction

- DW-GAM interaction.
  - Spontaneously excited by Drift Wave turbulence via 3-wave parametric resonant interactions
  - Scatter DW from unstable long wavelength to stable short wavelength
- Threshold of the parametric excitation determined by collisionless damping rate of GAM

  $\Rightarrow$ Need an analytical expression for a broad range of tokamak parameter, i.e., q, $k_r\rho$...

Previous theoretical work $k_r\rho_{it}q^2 << 1$

- Small magnetic drift orbit for resonant ions
  $k_r\rho_{d,res} \approx k_r\rho_{i,res}q < 1 \Rightarrow \omega_{d,res} < \omega_{t,res}$[Z. Gao, Sugama]

$$\delta H_g = \frac{qF_o\overline{\delta\phi}}{T} J_0(k_r\rho_i)\Sigma i^{l-p} e^{i(l-p)\theta} \frac{\omega J_l(k_r\rho_d)J_p(k_r\rho_d)}{\omega + l\omega_t} \quad l = \text{integer}$$

- Only low order harmonics of transit resonances contribute!
- $k_r\rho_{it}q^2 << 1 \Rightarrow l = \pm 1, \pm 2$ resonances[Sugama]

$$\begin{aligned} \gamma &= -\frac{\sqrt{\pi}}{2} q \frac{v_{it}}{R_0}[1 + \frac{2(23/4 + 4\tau + \tau^2)}{q^2(7/2 + 2\tau)^2}]^{-1} \times \\ &\times \left\{ \exp\{-\hat{\omega}_G^2\}\{\hat{\omega}_G^4 + (1+2\tau)\hat{\omega}_G^2\} + \exp\{-\hat{\omega}_G^2/4\} \right. \\ &\left. \times \frac{1}{4}(k_r\rho_{it}q)^2\{\frac{\hat{\omega}_G^6}{64} + (1 + \frac{3}{8}\tau)(\frac{\hat{\omega}_G^4}{8} + \frac{3\hat{\omega}_G^2}{4})\}\right\} \quad (1) \\ \hat{\omega}_G &= \frac{\sqrt{7+4\tau}}{2} q \left[1 + \frac{2(23 + 16\tau + 4\tau^2)}{q^2(7+4\tau)^2}\right]^{1/2} \quad (2) \end{aligned}$$

previous work $k_r \rho_{it} q^2 << 1$

- $k_r \rho_i q^2 \gg 1$
  - GAM exists in $q \gg 1$ regime to minimize Landau damping
  - 3-wave parametric excitation of GAM increases with $k_r \rho_{it}$
- higher-order harmonics of transit resonance contribute significantly

  $\Rightarrow$ No analytical formula for GAM collisionless damping rate!
- goal of this work $\Rightarrow$ analytical expression of GAM dispersion relation for $k_r \rho_{it} q^2 \ll 1$.

Linear gyrokinetic equation, Quasineutrality condition

- perturbed particle distribution function

$$\delta f = -eF_0\delta\phi/T + \exp[im_i c/(eB^2)\mathbf{k}\times\mathbf{B}\cdot\mathbf{v}]\delta H_g$$

- linear gyrokinetic equation

$$(\omega - \omega_d + i\omega_t\partial_\theta)\delta H_g = \frac{e}{T}F_0 J_0(k_\perp\rho_L)\omega\delta\phi \tag{3}$$

- electrons: $k\rho_e \ll 1$ and $\omega/\omega_{te} \sim \sqrt{m_e/m_i} \ll 1$

$$\delta f_e = e(\delta\phi - \overline{\delta\phi})F_{0e}/T_e$$

- quasineutrality condition

$$\frac{e}{T_e}(\delta\phi - \overline{\delta\phi}) = -\frac{e}{T_i}\delta\phi + \langle J_0\delta H_{gi}/N_0\rangle \tag{4}$$

non-resonant ion response

- determine the mode structures and real frequency
- expand in terms of the smallness parameters: $k_r\rho_{it}$ and $1/q$:
  - $\delta H_{g,nr} = \delta H^{(0)}_{g,nr} + \delta H^{(1)}_{g,nr} + \delta H^{(2)}_{g,nr} + \cdots$
  - $\delta\phi = \overline{\delta\phi} + \widetilde{\delta\phi}^{(1)} + \widetilde{\delta\phi}^{(2)} + \cdots$

$$\delta H^{(0)}_{g,nr} = \frac{e}{T}F_0 J_0 \overline{\delta\phi}; \qquad (5)$$

$$\omega\delta H^{(1)}_{g,nr} - (\omega_d - i\omega_t\partial_\theta)\delta H^{(0)}_{g,nr} = \frac{e}{T}F_0 J_0 \omega\widetilde{\delta\phi}^{(1)}; \qquad (6)$$

$$\omega\delta H^{(2)}_{g,nr} - (\omega_d - i\omega_t\partial_\theta)\delta H^{(1)}_{g,nr} = \frac{e}{T}F_0 J_0 \omega\widetilde{\delta\phi}^{(2)}; \qquad (7)$$

$$\vdots = \vdots$$

real frequency and mode structure

$$\begin{aligned}\widetilde{\delta\phi} = & - \left[1 - b(\frac{3}{2} + \tau) + (\frac{\tau}{2} + 1)\frac{\omega_{tt}^2}{\omega^2}\right] \tau \frac{\omega_{dt}}{\omega} \sin\theta \overline{\delta\phi} \\ & - \left[\frac{7}{4} + \tau - b(\frac{13}{4} + \frac{19}{4}\tau + 2\tau^2)\right] \tau \frac{\omega_{dt}^2}{2\omega^2} \cos 2\theta \overline{\delta\phi} \\ & - \left[\frac{9}{4} + \frac{7}{8}\tau - (\frac{9}{4} + \frac{7}{4}\tau + \frac{1}{2}\tau^2)\cos 2\theta\right] \tau \frac{\omega_{dt}^3}{\omega^3} \sin\theta \overline{\delta\phi}\end{aligned}$$

$$\begin{aligned}\omega_r = & (\frac{7}{4} + \tau)^{1/2} \frac{v_{it}}{R_0} \left\{1 + \frac{1}{2q^2}(\frac{23}{8} + 2\tau + \frac{\tau^2}{2})(\frac{7}{4} + \tau)^{-2} \right. \\ + & \left. \frac{b}{2}(\frac{7}{4} + \tau)^{-2}(\frac{1277}{64} + \frac{293}{32}\tau + \frac{3}{8}\tau^2 - \frac{\tau^3}{2})\right\}.\end{aligned}$$

resonant ions and dispersion relation imaginary part

- determine the collisionless damping
- expansion parameter:
  $|\omega_{t,res}/\omega_{d,res}| \sim |\omega_{t,res}/\omega| \sim 1/(k_r\rho_{it}q^2)^{1/2} \ll 1$

$$(\omega_d-\omega)\,\delta H^{(0)}_{g,res} = -\frac{e}{T_i}J_0F_0\omega\overline{\delta\phi}; \tag{8}$$

$$(\omega_d-\omega)\,\delta H^{(1)}_{g,res} = i\omega_t\frac{\partial}{\partial\theta}\delta H^{(0)}_{g,res} - \frac{e}{T_i}F_0\omega\widetilde{\delta\phi}^{(1)}; \tag{9}$$

$$(\omega_d-\omega)\,\delta H^{(2)}_{g,res} = i\omega_t\frac{\partial}{\partial\theta}\delta H^{(1)}_{g,res} - \frac{e}{T_i}F_0\omega\widetilde{\delta\phi}^{(2)}; \tag{10}$$

- D_i

$$D_i = \mathbb{I}\mathrm{m}\langle(J_0\overline{\delta H_{g,res}})\rangle/(\frac{eN_0}{T_i}\overline{\delta\phi}). \tag{11}$$

lowest order

- lowest order perturbed distribution function:

$$\delta H^{(0)}_{g,res} = -\frac{e}{T_i} J_0 F_0 \frac{\omega}{\omega_d - \omega} \overline{\delta\phi}. \tag{12}$$

- note:

$$\mathbb{I}\mathrm{m} \int \frac{d\theta}{2\pi} \frac{1}{\omega_d - \omega} = \int \frac{d\theta}{2\pi} \frac{\pi\delta(\theta - \theta_{res})}{\hat{\omega}_d \cos\theta}; \tag{13}$$

- $D^{(0)}_i$, the $q \to \infty$ limit:

$$D^{(0)}_i = \sqrt{2} \frac{\omega}{|\omega|} \left(1 + \frac{\omega^2_{dt}}{\omega^2} - 2b\right) \exp\left\{-\sigma\omega/\omega_{dt}\right\} \tag{14}$$

We need to go to higher orders to include q dependence.

lowest order correction

- $$\delta H^{(1)}_{g,res} = \frac{\partial}{\partial\theta}\left(\frac{i\omega_t\omega S}{2(\omega_d-\omega)^2}\right) - \frac{e}{T_i}\frac{\omega F_0\widetilde{\delta\phi}^{(1)}}{\omega_d-\omega}, \quad (15)$$
- lowest order correction to D_i

$$D^{(1)}_i = \sqrt{2}\left(1-\sigma\frac{\omega_{dt}}{2\omega}\right)\sigma\tau\frac{\omega_{dt}}{|\omega|}\exp\{-\sigma\omega/\omega_{dt}\} \quad (16)$$

No q dependence in $D^{(1)}_i$, need to go to next order to find q dependence.

second order correction, q dependence

- $$\delta H^{(2)}_{g,res} = \frac{i\omega_t}{\omega_d-\omega}\frac{\partial^2}{\partial\theta^2}\left(\frac{i\omega_t\omega S}{2(\omega_d-\omega)^2}\right) - \frac{e}{T_i}F_0\frac{\omega}{\omega_d-\omega}\widetilde{\delta\phi}^{(2)} + \frac{i\omega_t}{\omega_d-\omega}\frac{\partial}{\partial\theta}\left(\frac{e}{T_i}F_0\frac{\omega}{\omega_d-\omega}\widetilde{\delta\phi}^{(1)}\right) \quad (17)$$

- second order correction to D_i

$$D^{(2)}_i = \frac{\sqrt{2}}{24}\omega\omega^2_{tt}\left(-\frac{4\sigma}{\omega^3_{dt}}+\frac{\omega}{\omega^4_{dt}}\right)\exp\left\{-\sigma\frac{\omega}{\omega_{dt}}\right\} + \sqrt{2}\left(\frac{7}{4}+\tau\right)\frac{\omega^2_{dt}}{\omega^2}\tau\exp\left\{-\sigma\frac{\omega}{\omega_{dt}}\right\}, \quad (18)$$

Does contain $1/q^2$ correction in ω^2_{tt}!

compare with numerical results

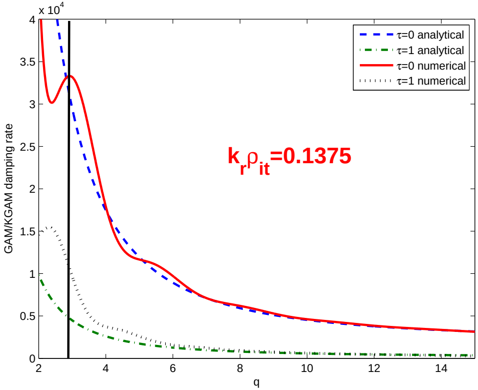

Figure: GAM/KGAM damping rate vs q. The four curves are respectively the numerical and analytical damping rate with $\tau = 0$ and 1.

Conclusion

- Derived analytical expression for GAM real frequency for $1 \gg k_r \rho_{it}$ and $1 \gg 1/q$ with $O(k^2 \rho_{it}^2)$ and $O(1/q^2))$ corrections;
- Derived analytical expression for GAM damping rate for $1 \gg k_r \rho_{it} \gg 1/q^2$;
- Combining with the previously derived damping rate expression for $1 \gg 1/q^2 \gg k_r \rho_{it}$, GAM damping rate over a broad range of tokamak parameters is determined.

Thank You!