Global particle-in-cell simulations of Alfvénic modes

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Keywords: gyrokinetic, global particle-in-cell, Alfvén Eigenmodes, fast-particle destabilization, Energetic Particle Modes

Acknowledgments: J. Nührenberg, P. Helander

Role of the fast particles

- **Fusion born fast particles (3.5 MeV), NBI (0.1 MeV), ICRH (1 MeV):** characteristic time scales (e.g. transit frequency ω_t) $\sim \omega_{TAE} \Rightarrow$
$\Rightarrow$ effective resonant interaction/destabilization

- **Bad news:** outward transport of fast particles $\Rightarrow$ non-even heat load on the wall + possible quenching of the fusion reaction

- **Good news:** weak destabilization $\Rightarrow$ MHD spectroscopy, alpha particle channeling (direct transfer the energy of fusion alphas into ions without intermediate step of slowing down on thermal electrons)

ALFVÉN MODE DYNAMICS IS IMPORTANT FOR FUSION

Gyrokinetic Vlasov-Maxwell equations

- **Linearized gyrokinetic Vlasov equation:**

$$\frac{\partial \delta f_s}{\partial t} + \dot{\vec{R}}^{(0)} \cdot \frac{\partial \delta f_s}{\partial \vec{R}} + \dot{v}_\parallel^{(0)} \frac{\partial \delta f_s}{\partial v_\parallel} = -\dot{\vec{R}}^{(1)} \cdot \frac{\partial F_{0s}}{\partial \vec{R}} - \dot{v}_\parallel^{(1)} \frac{\partial F_{0s}}{\partial v_\parallel}$$

- **Gyrocenter equations of motion ($p_\parallel$-formulation):**

$$\dot{\vec{R}} = \left(v_\parallel - \frac{q}{m} \langle A_\parallel \rangle \right) \vec{b}^* + \frac{1}{q B_\parallel^*} \vec{b} \times \left[\mu \nabla B + q \left(\nabla \langle \phi \rangle - v_\parallel \nabla \langle A_\parallel \rangle \right) \right]$$

$$\dot{v}_\parallel = -\frac{1}{m} \left[\mu \nabla B + q \left(\nabla \langle \phi \rangle - v_\parallel \nabla \langle A_\parallel \rangle \right) \right] \cdot \vec{b}^*$$

- **The gyro-averaged potentials are defined as usual:**

$$\langle \phi \rangle = \oint \frac{\mathrm{d}\theta}{2\pi} \phi(\vec{R} + \rho) \ , \quad \langle A_\parallel \rangle = \oint \frac{\mathrm{d}\theta}{2\pi} A_\parallel(\vec{R} + \rho)$$

Gyrokinetic Vlasov-Maxwell equations

- **Gyrokinetic quasineutrality equation and parallel Ampére's law ($p_\|$-formulation):**

$$-\nabla\cdot\left[\left(\sum_{s=i,f}\frac{q_s^2 n_s}{T_s}\rho_s^2\right)\nabla_\perp\phi\right]=\sum_{s=i,e,f}q_s\delta n_s$$

$$\left(\sum_{s=i,e,f}\frac{\hat{\beta}_s}{\rho_s^2}-\nabla_\perp^2\right)A_\|=\mu_0\sum_{s=i,e,f}\delta j_{\|s}$$

- **The gyrocenter perturbed density and current:**

$$\delta n_s=\int \mathrm{d}^6Z\,\delta f_s\,\delta(\vec{R}+\rho-\vec{x})\ ,\quad \delta j_{\|s}=q_s\int \mathrm{d}^6Z\,\delta f_s\,v_\|\,\delta(\vec{R}+\rho-\vec{x})$$

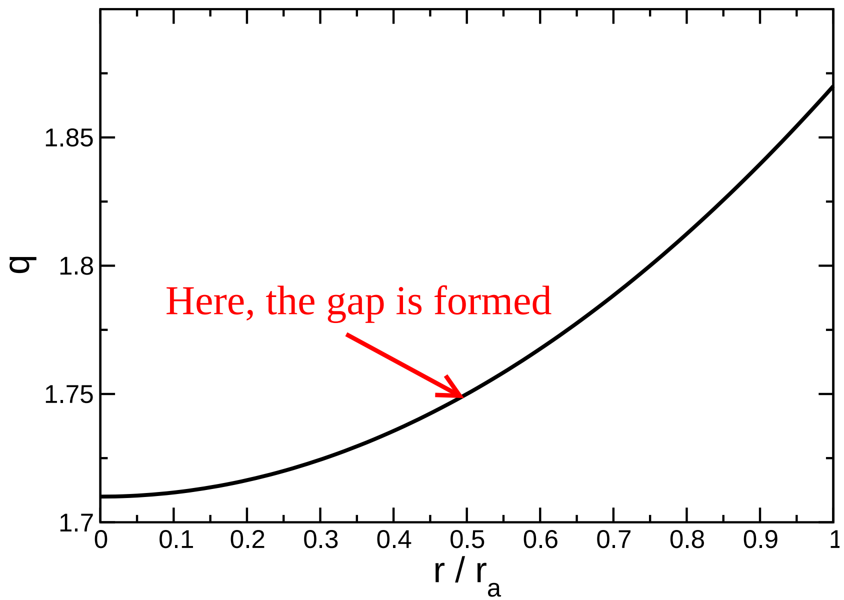
- **The background densities satisfy the quasineutrality equation**

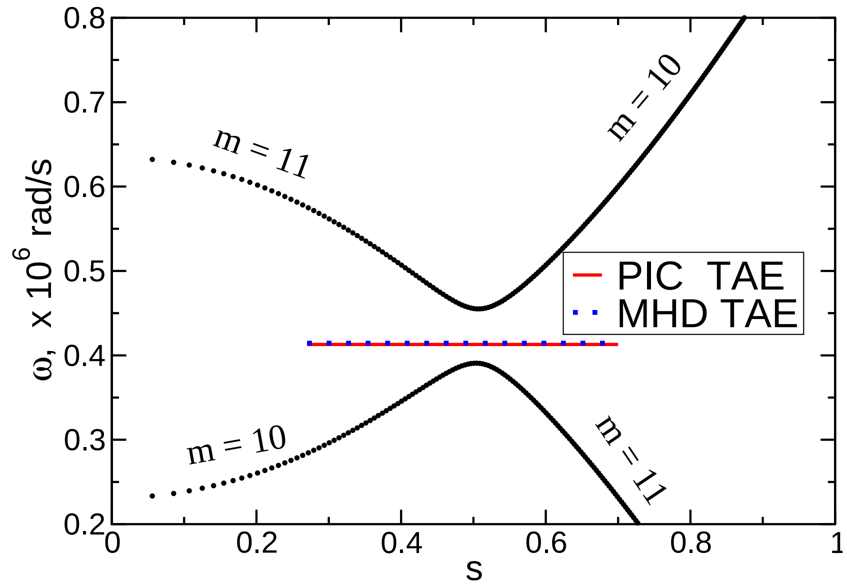
$$\sum_{s=i,e,f}q_s n_s=0$$

Numerical approach

- linear δf- particle-in-cell (PIC) method
- field discretization using B-splines
- phase factor transform
- Fourier transform in the direction of symmetry
- most serious numerical problem to solve for electromagnetic calculations: cancellation problem
- iterative solution of Ampere's law to cancel unphysical "adiabatic currents" detailed description:
  R. Hatzky, A. Könies, and A. Mishchenko, J. Comp. Phys. 255, 568 (2007)
  Similar to Y. Chen and S. Parker approach
- Performance optimization: parallel efficiency 97%, 4096 cores, Blue Gene/P (weak scaling)

Tokamak configuration





Large-aspect-ratio, circular cross-sections

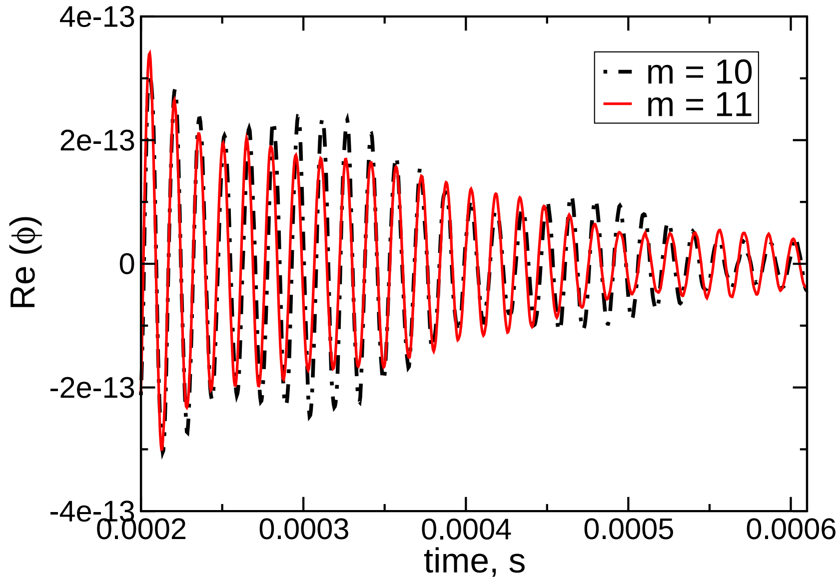
Major radius $R_0 = 10$ m, minor radius $r_a = 1$ m

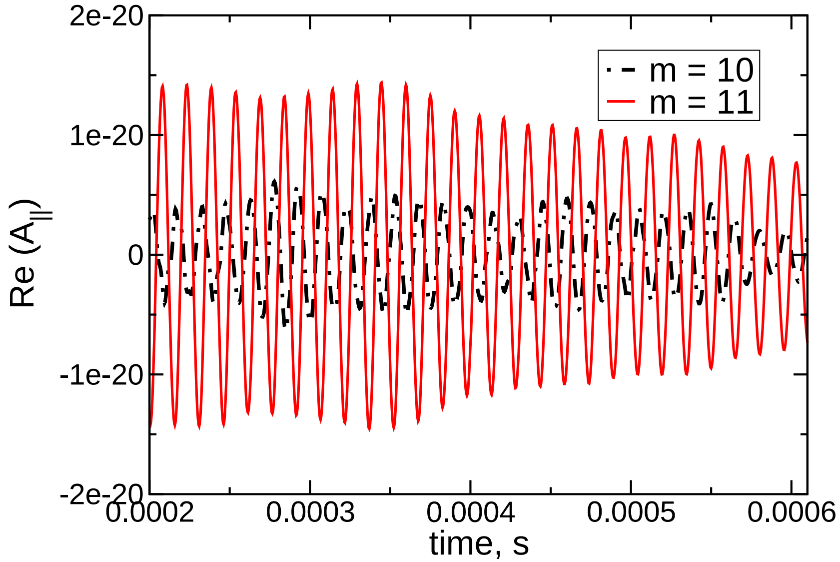
Magnetic field on the axis $B_0 = 3.0$ T,

Flat bulk-plasma temperature and density ($\beta_{\mathrm{bulk}} \approx 0.18\%$)

Toroidal mode number $n = 6$

Time signal

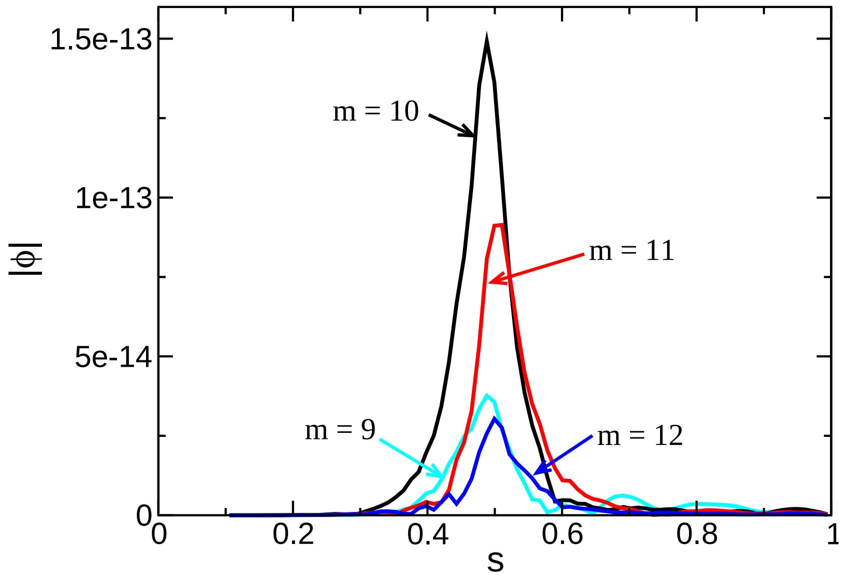


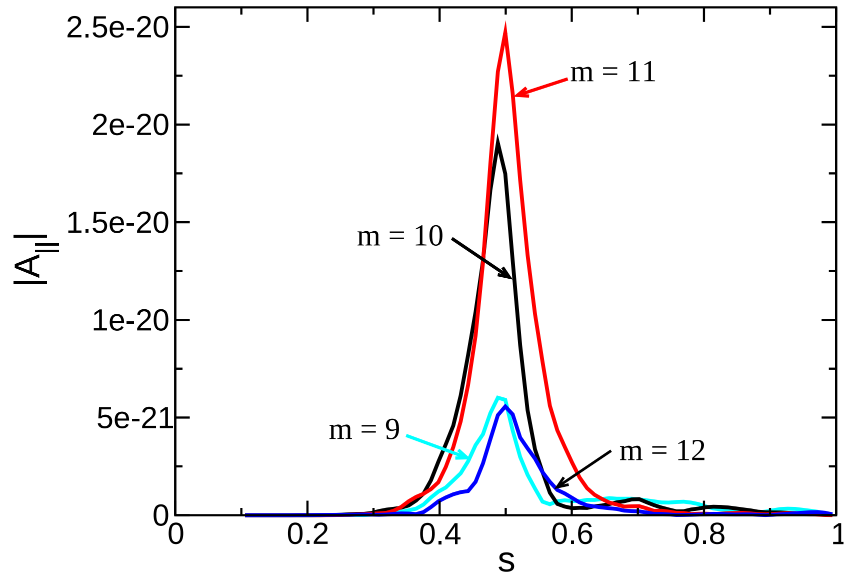


Time signal resulting from the PIC simulations.

Dominant harmonics in ϕ have the same phase.
Dominant harmonics in $A_{||}$ have the opposite phase.
This corresponds to the property $E_{||} \approx 0$.

Radial structure

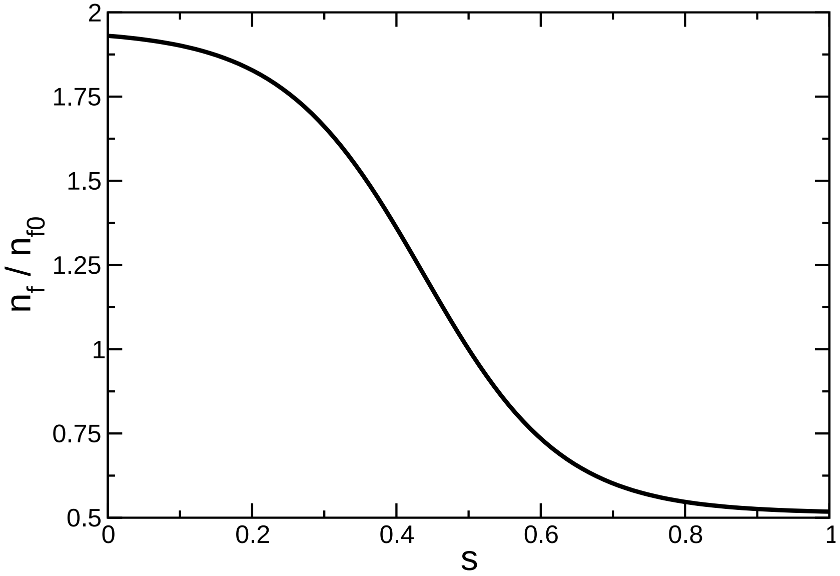


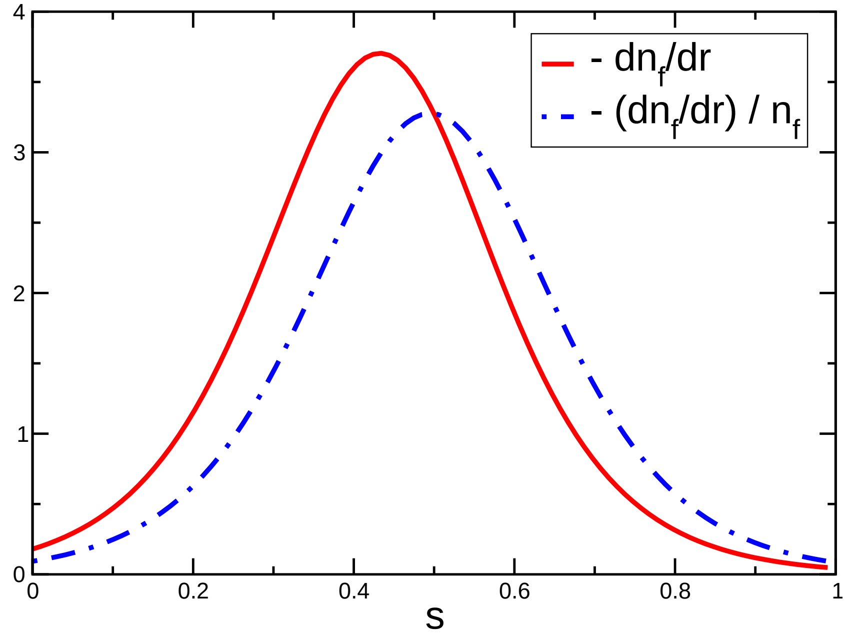


Radial pattern resulting from the PIC simulations (in some particular point of time)

It resembles a typical TAE structure.

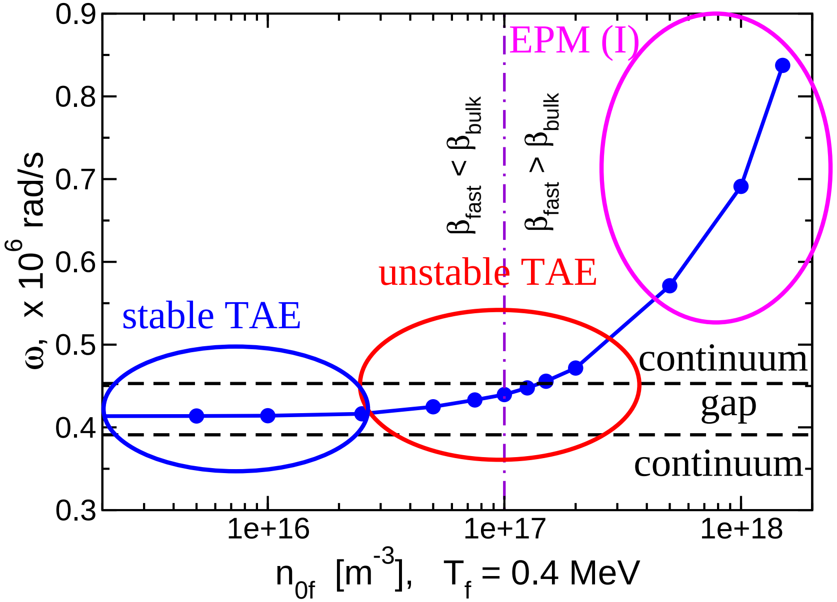
The fast-particle profiles

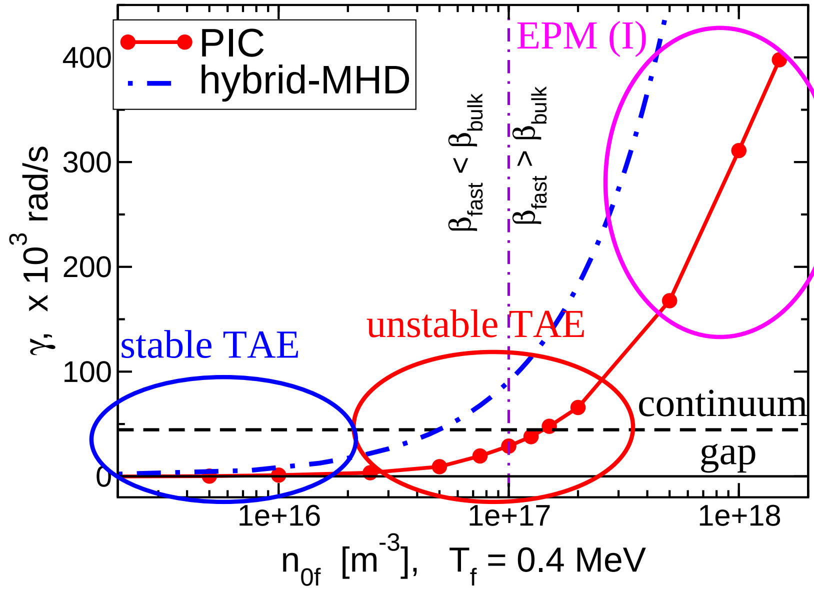




Nonuniform fast-particle density is used to drive the modes
Position of max. $\mathrm{d}\ln n_f/\mathrm{d}r$ coincides with the position of the gap
The fast-particle temperature is flat

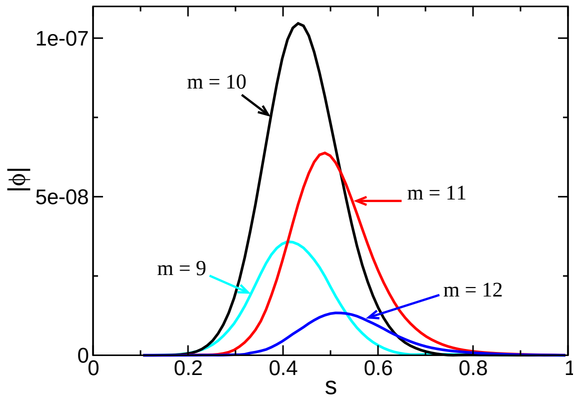
Fast-particle density sweep

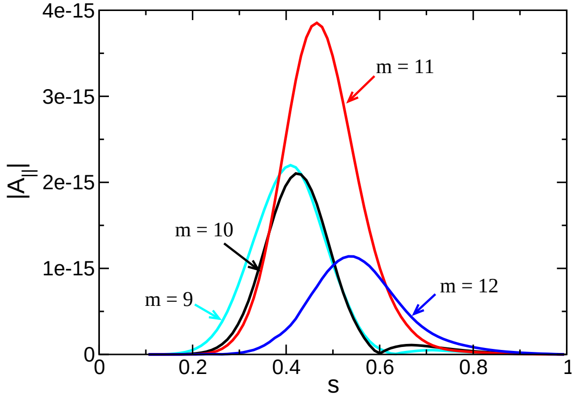




TAE destabilized by fast particles. It is continuously modified into EPM as the drive increases.
Hybrid-MHD calculations (CAS3D-K) overestimate the growth rate (FLR and FOW are neglected in CAS3D-K)

Energetic Particle Mode (Type I)



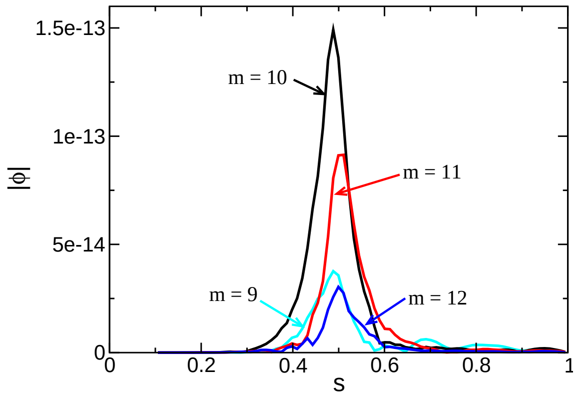


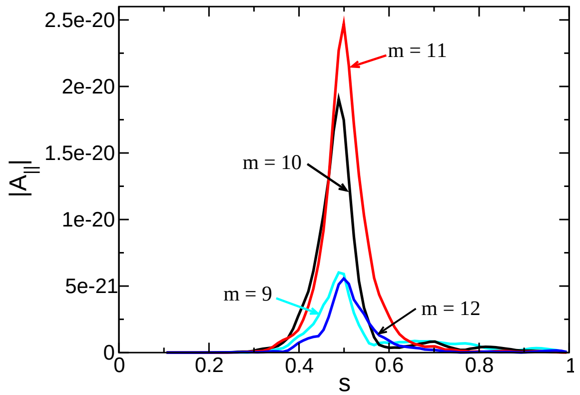
EPM (Type I)

$\beta_f \approx 1.8\%$

$n_f = 10^{18}\ m^{-3}$

$T_f = 0.4\ MeV$



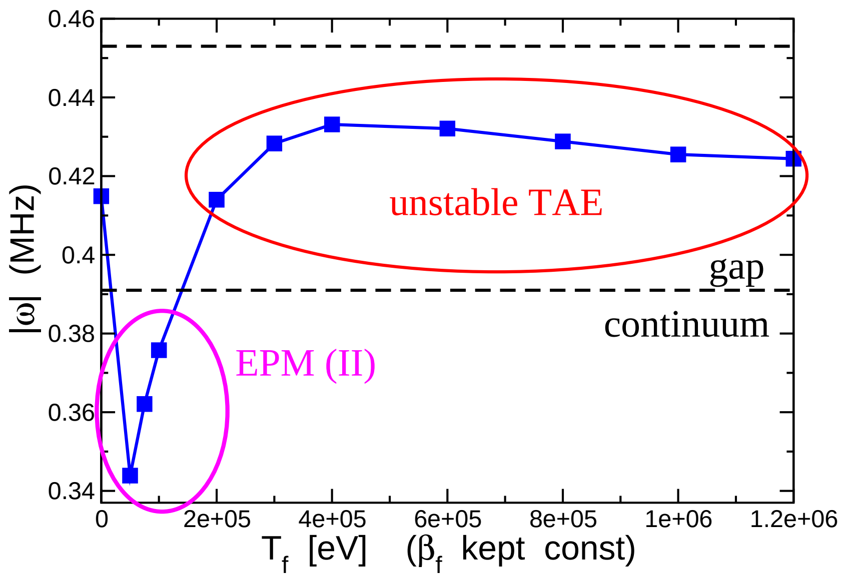


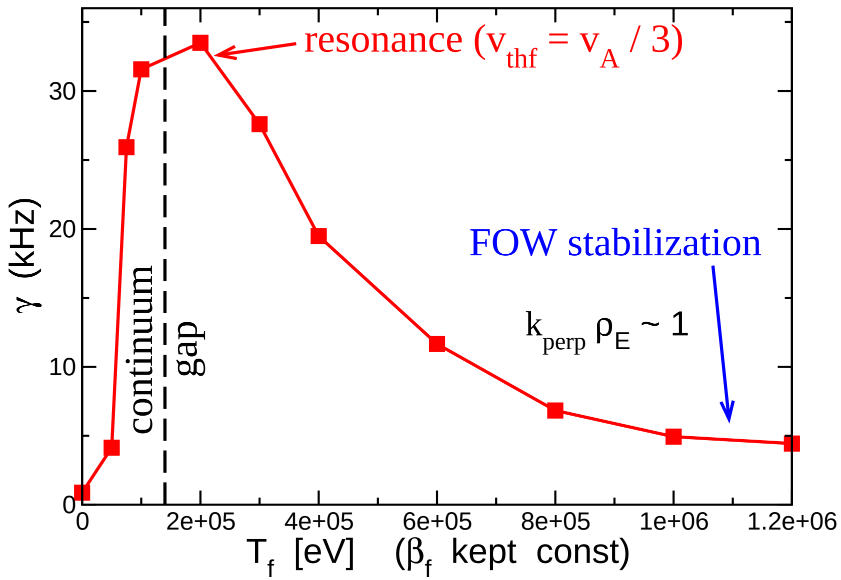
stable TAE

(no fast particles)

$\beta_f = 0$

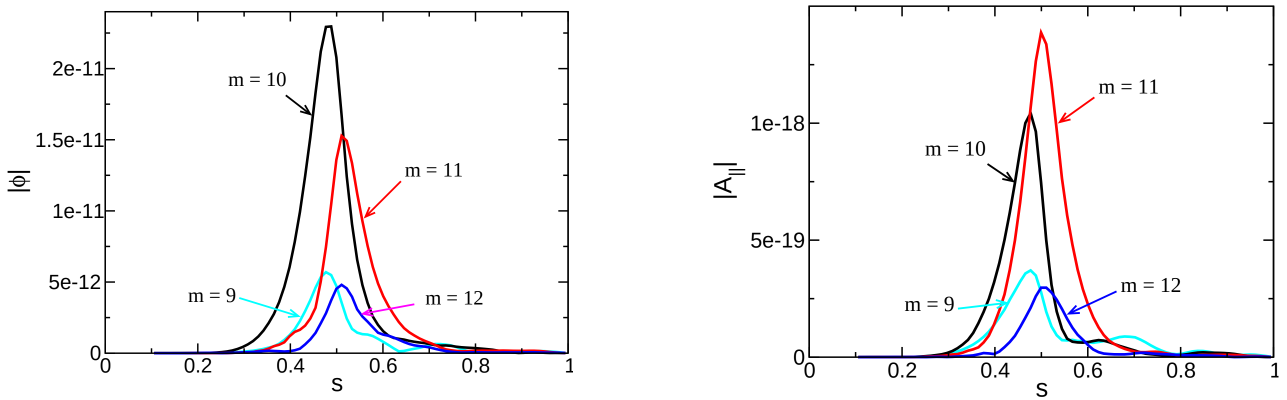
Fast-particle temperature sweep (I)





Dependency on the fast-particle temperature ($\beta_f = 0.134\%$ kept constant)
Destabilization is most effective near the resonance $v_{thf} \approx v_A/3$
At large T_f, finite-orbit-width (FOW) stabilization is seen
At smaller T_f (larger n_f to keep β_f constant), an EPM appears

Unstable Toroidal Alfvén Eigenmode

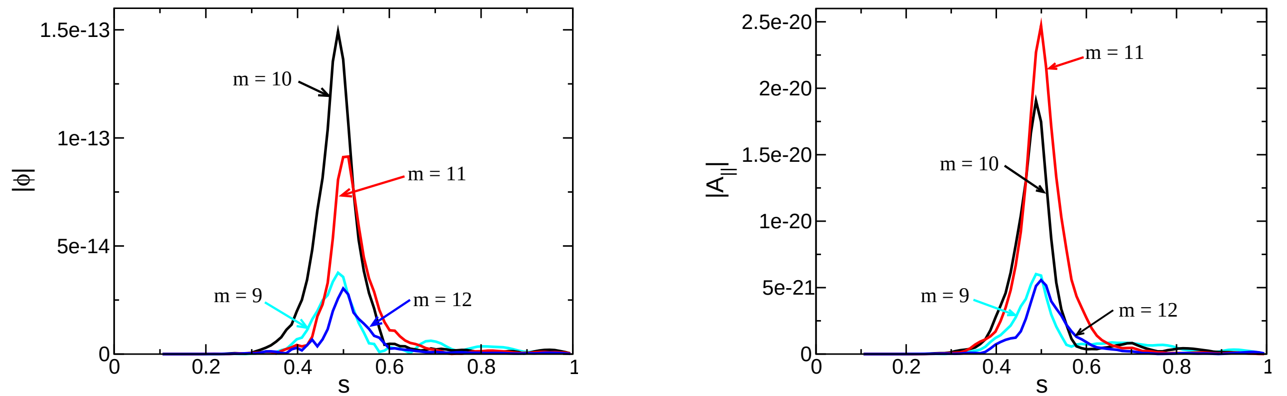


unstable TAE

$\beta_f \approx 0.134\%$

$n_f = 0.5 \times 10^{17}\ m^{-3}$

$T_f = 0.6\ MeV$

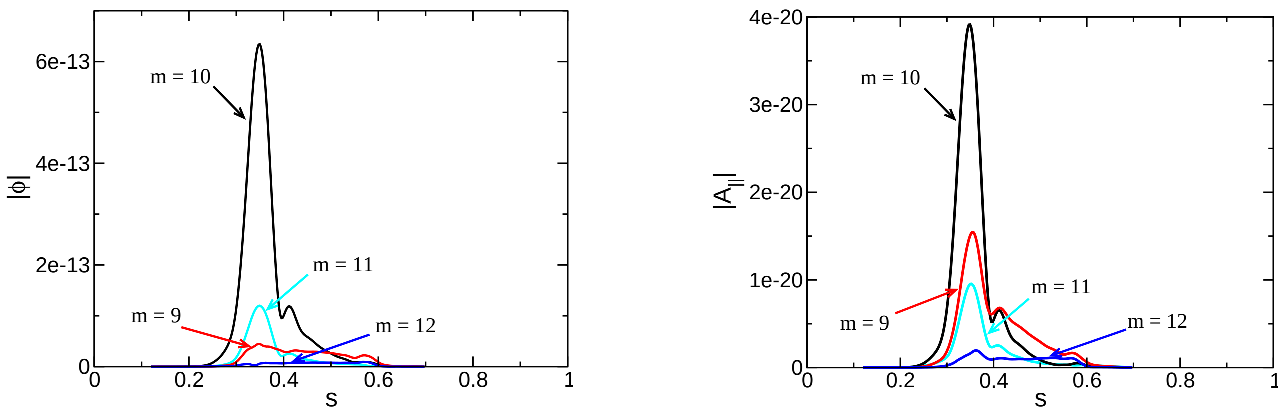


stable TAE

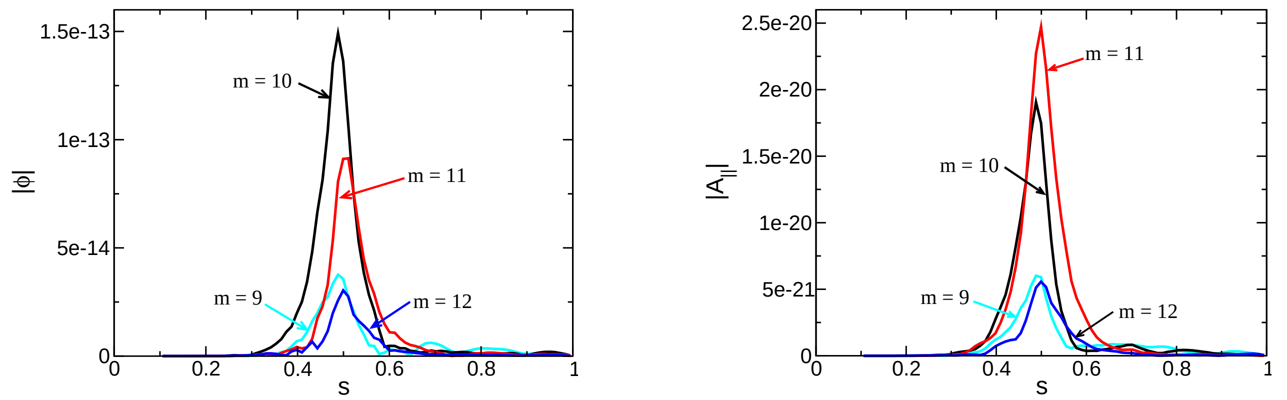
(no fast particles)

$\beta_f = 0$

Energetic Particle Mode (Type II)

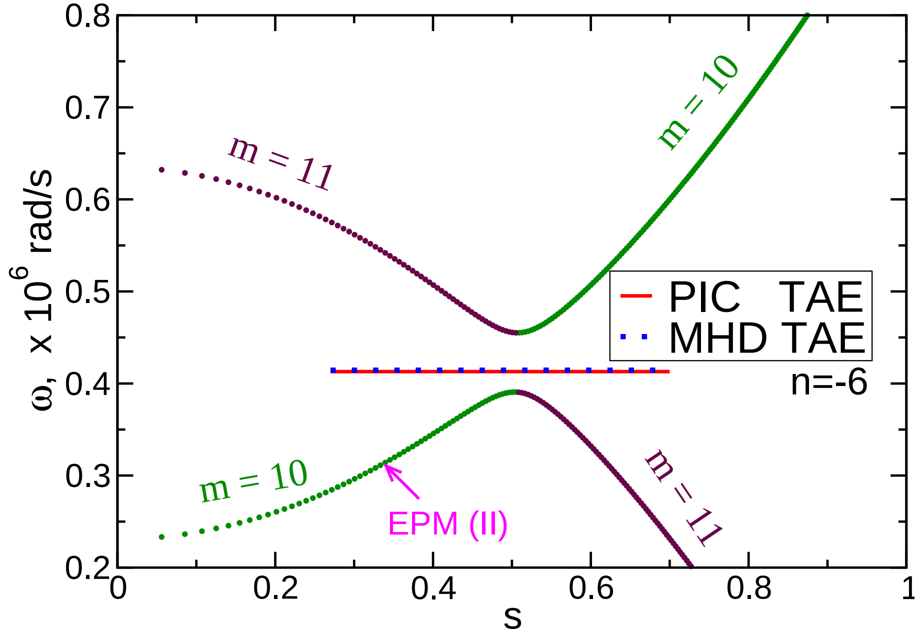


EPM (Type II)
$\beta_f \approx 0.134\%$
$n_f = 6 \times 10^{17}\ m^{-3}$
$T_f = 0.05\ MeV$



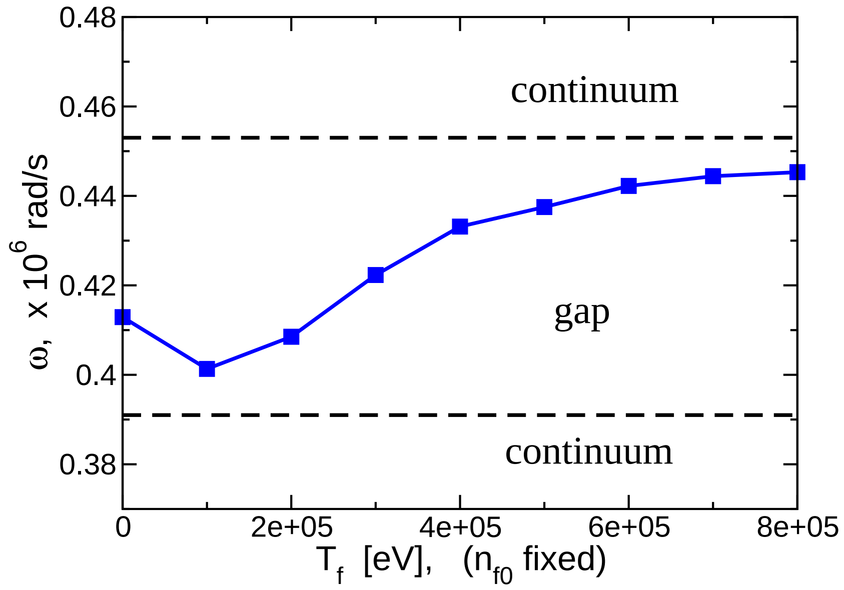
stable TAE
(no fast particles)
$\beta_f = 0$

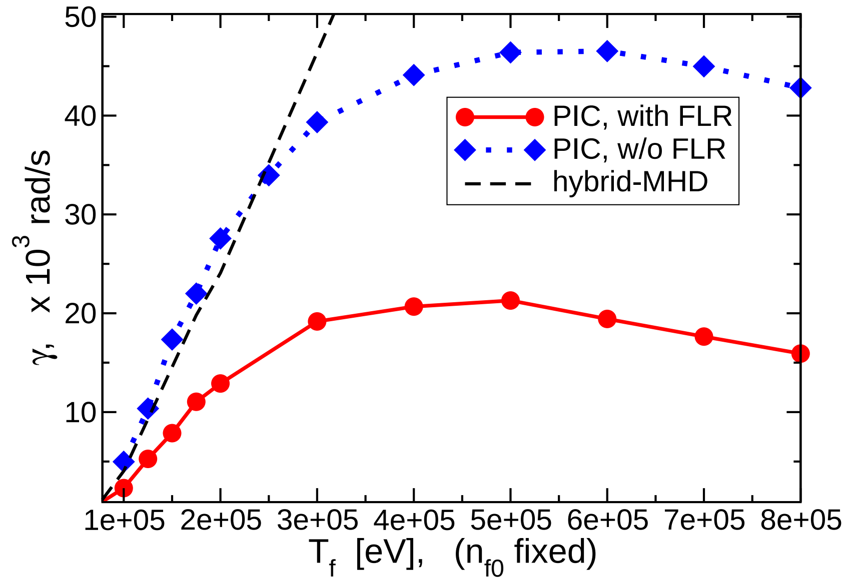
Energetic Particle Mode (Type II)



Location and frequency of the EPM mode are determined by continuum

Fast-particle temperature sweep (II)





The fast-particle density $n_f = 0.75 \times 10^{17}\ m^{-3}$ is kept constant
The mode frequency remains in the gap (no modification into the EPM)
At larger temperature, FOW stabilization can be seen

Fast particles. Unstable Alfvén modes. Summary.

- **Fast-particle destabilization of the TAE modes has been modelled with PIC code**
- **A transformation of the TAE mode into the EPM instability if the drive is large enough has been observed**
- **Next steps**
  - **Benchmarking: ORB5, LIGKA, GENE (global), GTC (?)**
  - **Numerical equilibria, smaller aspect ratio (more reactor-like)**

Global particle-in-cell simulations of Alfvénic modes

- **Alfvén-sound couplings (BAE ...), Alfvén cascades**
- **Kink mode, microtearing mode**
- **Nonlinear effects**
  - **nonlinear TAE/EPM phase-space dynamics (avalanches, spontaneous hole-clump pair creation, etc)**
  - **nonlinear TAE-EPM saturation**
- **Alfvén Eigenmodes + fast particles in stellarators (MAE, GAE, HAE etc)**