"Resonant and non-resonant particle dynamics in Alfvén mode excitations"

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(*covered in this session ...*)

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1 Motivation, essence

What is studied?

- (in)stability of <u>all</u> Alfvénic fluctuations which may potentially affect transport and confinement energetic ions

Why?

- α particles (fusion products) and energetic Deuterium and Tritium ions must be contained efficiently for self-sustained fusion to be feasible (self-heating plasma)

How?

- derivation of the eigenmode equations which contain all the essential mechanisms
  $\rightarrow$ <u>done in this paper</u>
- derivation of a plasma dispersion function in a reduced model
  $\rightarrow$ <u>done in this paper</u>
- numerical solution of the linear eigenmode equations
  $\rightarrow$ next step (my job)

Particle species

- <u>Thermal ions ($s = \mathrm{i}$)</u>: small/finite $k_\perp\rho_\mathrm{L} < 1$, carry plasma mass, provide inertia for Alfvén waves
- <u>Energetic ions ($s = \mathrm{E}$)</u>: large $k_\perp\rho_\mathrm{L}$ ($\lesssim 1$ ... $\gg 1$), relatively small fraction of plasma mass, carry lots of plasma energy ($\beta_\mathrm{E} \propto P_\mathrm{E}$, $\alpha_\mathrm{E} \propto P'_\mathrm{E}$)
- <u>Electrons ($s = \mathrm{e}$)</u>: negligible $k_\perp\rho_\mathrm{L}$ and fraction of plasma mass, quasi-adiabatic response $\rightarrow$ quasi-neutrality (except for trapped electrons!)

Dynamic scales of interest

- <u>Spatial:</u>

$$\lambda_\mathrm{D} \ll \lambda_\perp \ll a \tag{1}$$

  $\rightarrow$ ignore macro-instabilities ($\lambda_\perp \sim a$) such as kink and tearing modes; use quasi-neutrality approximation ($\lambda_\mathrm{D} \ll \lambda_\perp$)

$$\lambda_\perp \ll \lambda_\parallel \qquad (k_\perp \gg k_\parallel) \tag{2}$$

  $\rightarrow$ flute-like fluctuations; use eikonal representation (separation of scales)

$$\lambda_\perp < \rho_\mathrm{L} \qquad \text{and} \qquad \lambda_\perp \gg \rho_\mathrm{L} \quad \text{covered} \tag{3}$$

  Note: for $\lambda_\perp \sim \rho_\mathrm{L}$ most effects covered, however, check numerically whether need to add terms $\propto \mathbf{B}_0 \cdot \nabla(k_\perp\rho_\mathrm{L})$ for $s = \mathrm{E}$
  $\rightarrow$ broad range of spatial scales relative to ρ_L can be covered

- <u>Temporal:</u>

$$\overline{\omega_\mathrm{di}} \ll \omega_\mathrm{bi} \ll \underbrace{\omega_{*\mathrm{i}} \sim \omega_\mathrm{ti} \sim \overline{\omega_\mathrm{dE}} \sim \omega_\mathrm{bE} \lesssim \omega_\mathrm{A} \sim \omega_\mathrm{tE}}_{\substack{\omega_\mathrm{KBM/BAE} \cdots \omega_\mathrm{TAE} \\ \omega_\mathrm{EPM}}} \ll \omega_{*\mathrm{E}} \tag{4}$$

- Present model includes: (*I promise a more detailed discussion in a future presentation*)
  - **Resonant interactions:** precession, precession-bounce, transit resonances
    → direct instability drive
  - **Non-resonant interactions:** real frequency shifts, non-eigenfrequency oscillations and instabilities, compression, charge uncovering (topic of Section 4)
    → contributes to instability drive by helping to balance the positive MHD potential energy (instability threshold)
- Further remarks: An extended version of the model can be used to study ...
  - KAW ⇔ energetic particles
    [inclusion of finite $k_\perp \rho_\mathrm{i}$ effects gives rise to kinetic Alfvén waves (KAW)]
  - trapped electron effects

2 Model equations

The derivation encompasses the components and manipulations:

- GKE (non-adiabatic response) + Maxwell's equations (pre-Maxwell form)
- constitutive relations (velocity moments): $\langle f \rangle \to n$, $\langle v f \rangle \to j$, and $\left\langle v^2 f \right\rangle \to P$
- gyro-averaging: $\delta\phi(\mathbf{r}_\mathrm{gc} + \boldsymbol{\rho}) \to \delta\hat{\phi}(\mathbf{r}_\mathrm{gc}) \exp\left(i\mathbf{k}_\perp \cdot \boldsymbol{\rho}(\xi)\right) \to \left\langle \delta\hat{\phi} \exp\left(ik_\perp \rho_\mathrm{L} \sin(\xi)\right)\right\rangle_\xi \to \delta\hat{\phi} J_0(\lambda)$
- for convenience: separation of drift-kinetic limit ($k_\perp \rho_\mathrm{L} \to 0$) and FLR corrections
  $J_0 \to 1 - (1 - J_0)$ with $(1 - J_0) \propto (k_\perp \rho_\mathrm{L})^2$

Linear gyrokinetic equation (GKE), Eq. (1): Evolution of the nonadiabatic response (δG)
→ more precisely, a conveniently defined auxillary distribution function δK

$$f = F_0 + \delta f, \qquad \delta f = (\text{adiabatic}) + \delta G, \qquad \delta G = (...) + \delta K \tag{5}$$

[Equation for δK → given in the Section 3]
In applications, an isotropic Maxwellian velocity distribution is used for F_0: $F_0 = n_0(r) F_\mathrm{M}(\mathcal{E})$.

Vorticity equation, Eqs. (3,7,13): charge density continuity + parallel Ampére's law + quasi-neutrality assumption

$$\underbrace{\partial_t^2 \delta\psi - \nabla_\parallel^2 \delta\psi}_{\text{SAW}} = \underbrace{[\omega_*] \times (\delta\psi, \delta B_\parallel)}_{\text{drift waves}} + \underbrace{\langle \delta K \rangle_s}_{\substack{\text{kinetic compression,}\\ \text{resonances}}} + \underbrace{[\text{FLR}] \times (\delta\phi, \delta\psi, \delta B_\parallel)}_{\text{FLR effects}} + \underbrace{[\text{FLR}, \partial_\mu F_0] \times (\delta\phi, \delta B_\parallel)}_{\text{ignored}} \tag{6}$$

[More in the Section 3]

Quasi-neutrality condition (QN), Eqs. (5,11,14):

$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}t} \sum_{s=\mathrm{e,i,E}} \langle e \delta f \rangle_s}_{\text{QN}\to 0} = \underbrace{[\partial_\mathcal{E} F_0] \times \delta E_\parallel + [\text{FLR}] \times \delta\phi + \langle e J_0 \delta K \rangle_s}_{\to\text{charge uncovering}} + \underbrace{[\text{FLR}, \partial_\mu F_0] \times (\delta\psi, \delta B_\parallel)}_{\text{ignored}} \tag{7}$$

Magnetic compression, Eqs. (6,12,15): perpendicular Ampére's law

$$\delta B_\parallel = [\omega_*] \times \delta\psi + [\text{FLR}] \times \delta\phi + [\text{FLR}] \times \langle \delta K \rangle_s + \underbrace{[\text{FLR}, \partial_\mu F_0] \times (\delta\phi, \delta B_\parallel)}_{\text{ignored}} \tag{8}$$

3 Vorticity equation

The vorticity equation is insightfully derived by starting from the zeroth velocity moment of the GKE, i.e., the **continuity equation for the perturbed charge density**:

$$\frac{i\omega\mu_0}{k_\vartheta^2}\sum_s \langle eJ_0(\lambda)\times(\mathrm{GKE})\rangle_s \rightarrow \sum_s (e_s\partial_t\delta n_s + \nabla\cdot\delta\mathbf{j}_s) \overset{\mathrm{QN}}{\approx} \sum_s \nabla\cdot\delta\mathbf{j}_s = 0 \tag{9}$$

Knowing the results, we can identify the components of the GKE as follows:

$$\underbrace{v_\parallel\partial_l\delta K}_{\mathrm{FLB}}\ \underbrace{-i\omega\delta K}_{\mathrm{inertia}}\ \underbrace{+i\omega_\mathrm{d}\delta K}_{\mathrm{KPC}} = i\frac{e}{m}QF_0\{\underbrace{J_0(\delta\phi-\delta\psi)}_{\delta E_\parallel}\ \underbrace{+\omega_\mathrm{d}J_0\delta\psi/\omega}_{\mathrm{MPC}}\ \underbrace{+(v_\perp/k_\perp)J_1\delta B_\parallel}_{\mathrm{MFC}}\} \tag{10}$$

where $\partial_l = \mathbf{B}_0\cdot\nabla$, $QF_0 = \omega\partial_\mathcal{E}F_0 + [(\mathbf{k}_\perp\times\hat{\mathbf{b}})\cdot\nabla F_0]/\omega_\mathrm{c}$ and

FLB	$\propto v_\parallel\partial_l\delta K$	:	field line bending
KPC	$\propto \omega_\mathrm{d}\delta K$	:	kinetic (non-MHD) non-adiabatic particle compression
MPC	$\propto \omega_\mathrm{d}\delta\psi$	:	MHD non-adiabatic particle compression
MFC	$\propto \delta B_\parallel$	:	magnetic field compression
ICU	$= -i\omega\delta K - (...)\delta E_\parallel$	:	inertia-charge uncovering

FLB term:

$$\sum_s \left\langle ev_\parallel\delta K\right\rangle_s \quad\rightarrow\quad \sum_s \left\langle ev\delta f\right\rangle_s + (...) \quad\rightarrow\quad \underbrace{\delta j_\parallel \approx \frac{1}{\mu_0}\nabla_\perp^2\delta A_\parallel}_{\text{parallel Ampére's law}} = \frac{k_\perp^2}{i\omega\mu_0}\partial_l\delta\psi \tag{11a}$$

$$\partial_l\sum_s \left\langle ev_\parallel\delta K\right\rangle_s \quad\rightarrow\quad \underbrace{\partial_l(k_\perp^2\partial_l\delta\psi)}_{\mathrm{FLB}} + \underbrace{(\text{kink term} \propto \nabla j_{0\parallel})}_{\text{ignored}} \tag{11b}$$

This term measures the amount of field line bending. By neglecting the kink term (due to focus on large wave numbers), we exclude instabilities driven by the current density gradient (e.g., kink and tearing modes).

ICU term: Combination of the inertia term and the term associated with $\delta E_\parallel$.

$$\sum_s \left\langle i\omega\delta K\right\rangle_s + [QF_0]\times\delta E_\parallel \quad\rightarrow\quad \underbrace{\frac{\mathrm{d}}{\mathrm{d}t}\sum_s\left\langle e\delta f\right\rangle_s}_{\mathrm{QN}\rightarrow 0} + [\mathrm{FLR}]\times\delta\phi \tag{12}$$

Here, "charge uncovering" refers to a lack of charge screening along the field line on a length scale greater than the Debye length. The finite electron mass, the finite ion Larmor radius and the large magnetic drift orbits of energetic ions are effects that prohibit instantaeous charge neutralization along **B** and may give rise to noticable effects associated with $\delta E_\parallel$. The charge uncovering concept is discussed further in Section 4.

4 Quasi-neutrality and charge uncovering

Let us throw some more light on the nature of the voricity equation and the charge uncovering concept. Consider once more the **gyrokinetic equation** (10), omitting terms $\propto \partial_l J_0$ for simplicity:

$$\underbrace{v_\parallel \partial_l \delta K}_{\text{FLB}} \underbrace{-i\omega\delta K}_{\text{inertia}} \underbrace{+i\omega_{\rm d}\delta K}_{\text{KPC}} = i\frac{e}{m}QF_0[\underbrace{J_0(\lambda)(\delta\phi - \delta\psi)}_{\delta E_\parallel} \underbrace{+(\omega_{\rm d}/\omega)J_0(\lambda)\delta\psi}_{\text{MPC}} \underbrace{+(v_\perp/k_\perp)J_1(\lambda)\delta B_\parallel}_{\text{MFC}}] \quad (13)$$

The velocity moment $(i\omega\mu_0/k_\vartheta^2)\sum_s \langle eJ_0\cdots\rangle_s$ turns this into the **charge continuity equation**,

$$\underbrace{\sum_s \mathbf{B}\cdot\nabla\left(\frac{\delta j_\parallel}{B}\right)_s}_{\text{(i)}} + \underbrace{\overline{\sum_s \nabla\cdot\delta\mathbf{j}_{\parallel s}}}_{\text{(ii)}} + \underbrace{\sum_s \frac{\partial}{\partial t}\langle e\delta f\rangle_s + \sum_s \delta\mathbf{v}_{\rm E}\cdot\nabla\langle eF_0\rangle_s}_{\text{(iii)}} = 0 \quad (14)$$

where we have defined $\overline{\sum_s \nabla\cdot\delta\mathbf{j}_{\parallel s}} = \sum_s \nabla\cdot\delta\mathbf{j}_{\parallel s} - \sum_s \delta\mathbf{v}_{\rm E}\cdot\nabla\langle eF_0\rangle_s$. Terms (i)–(iii) are linked to the gyrokinetic equation as follows:

- Term (i) is the remainder of the FLB term after dropping the kink term and the term $\propto \partial_l J_0$: $\frac{B}{k_\vartheta^2}\partial_l\left(\frac{k_\perp^2}{B}\partial_l\delta\psi\right) = \sum_s \frac{i\omega\mu_0 B}{k_\vartheta^2}\partial_l(\delta j_\parallel/B)_s$.
- Term (ii) contains KPC, MPC and MFC, and the FLR correction $[\propto (1 - J_0^2)]$ of the $\delta E_\parallel$ term.
- Term (iii) is obtained by combining the inertia term and the drift-kinetic limit of the $\delta E_\parallel$ term.

Vorticity equation. If carry out the summation over all species and assume quasi-neutrality, term (iii) vanishes and we obtain the vorticity equation, which may be compactly written as

$$\underbrace{\sum_s \mathbf{B}\cdot\nabla\left(\frac{\delta j_\parallel}{B}\right)_s}_{\text{FLB}} + \overline{\sum_s \nabla\cdot\delta\mathbf{j}_{\parallel s}} = 0 \quad (15)$$

Charge uncovering. Noting that Eq. (14) also <u>holds for each species individually</u>, we have for the energetic component

$$\underbrace{\mathbf{B}\cdot\nabla\left(\frac{\delta j_\parallel}{B}\right)_{\rm E}}_{\text{(i)}_{\rm E}} + \underbrace{\overline{\nabla\cdot\delta\mathbf{j}_{\perp\rm E}}}_{\text{(ii)}_{\rm E}} = \underbrace{-\frac{\partial}{\partial t}\langle e\delta f\rangle_{\rm E} - \delta\mathbf{v}_{\rm E}\cdot\nabla\langle eF_0\rangle_{\rm E}}_{-\text{(iii)}_{\rm E}} \overset{\text{QN}}{=} \underbrace{\sum_{s=\text{i,e}} \frac{\partial}{\partial t}\langle e\delta f\rangle_s + \sum_{s=\text{i,e}} \delta\mathbf{v}_{\rm E}\cdot\nabla\langle eF_0\rangle_s}_{\text{(iii)}_{\rm C}}$$

where the second equality follows from the quasi-neutrality assumption and "C" denotes "core" species ("e" and "i"). This may be interpreted as follows:

$$\underbrace{\mathbf{B}\cdot\nabla\left(\frac{\delta j_{\parallel\rm E}}{B}\right) + \overline{\nabla\cdot\delta\mathbf{j}_{\perp\rm E}}}_{\substack{\text{energetic ions}\\ \text{(large Larmor radii)}\\ \text{cause ...}}} \;\longrightarrow\; = \underbrace{\sum_{s=\text{i,e}}\left(\frac{\partial}{\partial t}\langle e\delta f\rangle + \delta\mathbf{v}_{\rm E}\cdot\nabla\langle eF_0\rangle\right)_s}_{\substack{\text{charge separation}\\ \text{between electrons and ions}\\ \text{(charge uncovering)}}} \;\longrightarrow\; = \underbrace{-\sum_{s=\text{i,e}}\left(\mathbf{B}\cdot\nabla\left(\frac{\delta j_\parallel}{B}\right) + \overline{\nabla\cdot\delta\mathbf{j}_\perp}\right)_s}_{\substack{\text{adiabatic response}\\ \text{of the core component}\\ \text{(quasi-neutrality)}}} \quad (16)$$

It is important to note that Eqs. (15) and (16) are completely equivalent. It can be seen that charge uncovering (CU) is a **non-resonant effect** associated with finite energetic ion density. In toroidal systems, CU is determined by magnetic drifts. The concept is useful in MHD theory, e.g., to interpret the core current induced by charge separation due to energetic ions.