Weak Turbulence Theory of Drift Waves

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Outline

* What's Weak Turbulence Theory ?

* Collisionless Trapped Electron Mode as an example

  - Review of Linear Properties
  - Ion Compton Scattering v.s. Trapped Electron Compton Scattering
  - Wave-kinetic Eqn : derivation & Solution.

* Homework.

Hierarchy of Microturbulence Theory

(in reverse order)

		Justification
No Theory	Life without perturbation	No curiosity
Linear Theory	1st order "	$\frac{\lvert e\rvert \delta\phi}{T_e}$, $\delta n / n_0 \ll 1$. infinitesimal.
Quasilinear Th	2nd order "	Slow evolution of $\langle f \rangle$
Weak Turbulence Theory	3rd order "	Dispersive Wave; $\lvert\delta\omega\rvert > \gamma_{lin}$
Strong Turbulence Theory	∞ - order, → "Renormalization"	In many applications, one should do this.

$$\begin{tikzcd}
\langle f \rangle \arrow[rr, bend left, "\text{linear Theory}"] & & \delta\phi_{\vec{k}}^{\text{linear}} \arrow[ll, bend left, "\langle f \rangle \text{ evolution}"]
\end{tikzcd}$$

Quasilinear theory.

Typically there are many unstable drift waves with different $\vec{k}$'s.

It's believed that $\phi_{\vec{k}}$ grows and saturates at finite amplitude mainly due to nonlinear couplings to other $\phi_{\vec{k}'}$'s, rather than due to $\langle f \rangle$ modification.

This nonlinear mode coupling can involve 3 different waves (3-wave coupling) or can be mediated by particles (nonlinear Landau damping or Compton Scattering).

Quick Review of (electron) Drift Waves:

(eg., GPP-II in Tang Dynasty, Chairman's Memorial Lecture, in Calif., Super Lin's Medal Ceremony in Beijing 2008)

- With ∇n only, $T_e \gg T_i$,

$$\omega \simeq \frac{\omega_{*e}}{1 + k_\perp^2 \rho_s^2}$$

: ω_{*e} from $E \times B$ advection of mean density gradient.

$$\equiv \frac{k_y \rho_s}{L_n} c_s .$$

$k_\perp^2 \rho_s^2$ from Polarization drift.

$v_{gr} = \frac{\partial \omega}{\partial k_y}$ varies with k_y $\Rightarrow$ dispersive.

Most ions satisfy $v_{i\parallel} < \frac{\omega}{k_\parallel}$ (fluid regime)

- Instability occurs due to electron (inverse) dissipation.

* What makes drift waves unstable ?

- Most e's move fast along $\vec{B}$, $\rightarrow$ described by Boltzmann rel$^{\underline{n}}$

$$\delta f \simeq \frac{|e|\delta\phi}{T_e} f_0 .$$

- The rest : nonadiabatic e's :

$$\left\{ \frac{\partial}{\partial t} + v_\parallel (\hat{b}\cdot\vec{\nabla}) \right\} \delta h_e = -\frac{|e|}{T_0}\frac{\partial}{\partial t}\delta\phi\, F_0 - c\frac{\hat{b}\times\nabla\delta\phi}{B_0}\cdot\vec{\nabla} F_0 ,$$

$$\propto i\,(\omega_{*e} - \omega)\frac{|e|\delta\phi}{T_e}$$

- $\mathrm{Im}\left\{ \int d^3v\, \delta h_e \right\}$

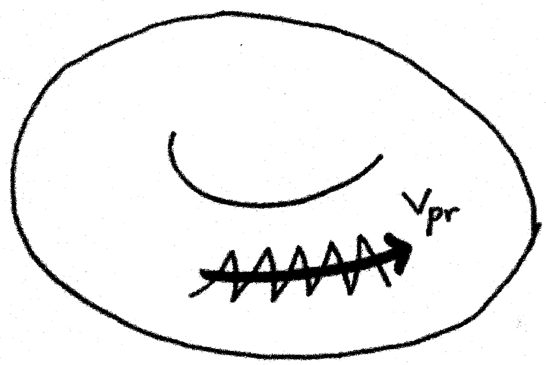
$$\Longrightarrow \gamma_{\vec{k}} \doteq -\frac{\pi T_e \omega_k}{2|k_\parallel| m_e}\left(\underbrace{\frac{k_\theta}{\Omega_e}\frac{\partial}{\partial x}}_{\substack{\text{Relaxation of}\\ \text{free energy in } \nabla n \\ \propto\, \omega_{*e}}} + \underbrace{k_\parallel \frac{\partial}{\partial v_\parallel}}_{\substack{\text{Electron heating}\\ \text{via relax}^{\underline{n}} \text{ in } f(v_\parallel) \\ \propto\, \omega}} \right) f_0 \Big|_{v_\parallel = \frac{\omega}{k_\parallel}}$$

Need downward-shift of ω below ω_{*e} for instability,

- **Properties of Collisionless Trapped Electron Modes in Torus**

Electron Drift Wave $(k_{\parallel} v_{Te} \gg \omega > k_{\parallel} v_{Ti})$
destabilized by trapped electron precession resonance

$$\omega_{be} > \omega \gtrsim \omega_{de} \equiv k_{\phi} V_{pr.} > \nu_{eff}.$$



$$\text{``} \mathrm{Im}\left(\frac{1}{\omega - \omega_{de}}\right) \text{''}$$

Resonance mechanism familiar from **"Fish-Bone"** driven by trapped ion precession resonance

$$\omega_{de} \cong \frac{L_n}{R} \omega_{*e} (E/T_e)$$

High energy trapped electron with $E/T_e \sim \frac{R}{L_n}$ **resonant.**

* **Instability becomes weaker as** R/L_n increases.

$$\gamma/\omega_{*e} \cong 2\pi^{1/2} \eta_e (2\epsilon)^{1/2} \left(\frac{R}{L_n}\right)^{3/2} \left(\frac{R}{L_n} - 3/2\right) e^{-R/L_n} \qquad \text{for } (k_\theta \rho_s \ll 1)$$

(Adam, Tang and Rutherford PF '76)